ANALITICAL METHOD TO CALCULATE THE UNKNOWN GEOMETRY OF CYLINDRICAL GEARS

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ABSTRACT

A procedure of reverse engineering to determine the basic geometry for manufacturing of external parallel-axis cylindrical involute gears by means of workshop measurement tools is presented. This procedure proposes a practical method to obtain the fundamental gear parameters in order to have a reference for calculating the load capacity of cylindrical gears or when a "copy" of an external parallel-axis cylindrical involute gear is necessary for recreating other new gear according to ISO standards by generation cutting.

KEYWORDS*: Cylindrical gear, unknown geometry, inverse engineering*

1.0 INTRODUCTION

Gear engineering requires of professional skills in several action fields like design, production, operation, maintenance, repair and recycled. Generally, the main action fields are established by the industry profile. Industries and companies with actions in the maintenance and repair of gear usually ask for professionals with skill in the recovery of these elements.

In general, the repair of gear implies bigger challenge to the gear engineers, because the problems and solutions involve already manufactured gears whose geometry is generally unknown. In this situation, the engineer needs to know the previous basic geometry of the gears in order to have a reference for the recovering or re-manufacturing.

Actually, there are a wide variety of CNC generative gear testers and coordinate measuring machines (CMM) destined for inspection and

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control of spur and helical gears with fully automatic measuring cycle and extremely short measuring times combined with high measuring accuracies. In this advanced gear measuring machines, the profile of the tooth can be checked and compared with a flank topography reference and by means of a trial and error procedure it is possible to obtain an approximate geometry of the analyzed gears (Kumar, 2014). Moreover, some advanced measurement machine have incorporated special program for measuring gears with unknown parameters and determining some important data of the gear basic geometry (Grimsley, 2003). Unfortunately, the price lists of these machines are very high, somewhere in the $300,000 to $500,000 range, and not often accessible to the company or factory involves with gear remanufacturing. Concerning with this situation, gear specialists (González Rey, 1999), (Innocenti, 2007), (Belarifi et al., (2008), (Schultz, 2010) involved with recreating replacement gears are considered alternative procedures to determine the unknown gear geometry using more simple measurement tools.

Consequently, this paper presents a method of reverse engineering to determine the unknown gear geometry in order to have a reference for the design or manufacturing. This method, based on author's experiences in the analysis, recovery and conversion of helical and spur gears, proposes a practical procedure with results not too exact, but practically acceptable, to obtain the fundamental parameters by means of conventional measurement tools. This method is useful for the recreating of new external parallelaxis cylindrical involute gears according to ISO standards by a generation cutting process.

2.0 BASIC GEAR DATA TO DETERMINE THE UNKNOWN EXTERNAL CYLINDRICAL INVOLUTE GEAR GEOMETRY

It is known that the question of what data is required to specify an external parallel-axis cylindrical involute gear can be answered perfectly by means of the theory associated with the involute helicoids surface of the flank of a helical gear (Maag, 1990). In this case, it is necessary to know number of teeth, tip diameter, root diameter, base diameter, base helix angle and base tooth thickness. The three first data can be determined easily by measurement but the data associated with the base cylinder can be determined only by special gear measuring equipment. Thus, where only a sample of a gear but not complete gear data is available initially, the specification for generating the gear can be calculated. Main formulas involved with the theory of the involute helicoids surface of the flank of a helical gear are summarized below (Equations (1) –(8)). Some of them are fundamentals in the determination of the gear geometry that fulfills the data requested as reference for the design or manufacturing.

$$d_f = \frac{m \cdot z}{\cos\beta} - 2 \cdot m \cdot \left(h_a^* + c^* - x\right) \tag{1}$$

$$d_{a_{1,2}} = 2 \cdot a_w - d_{f_{2,1}} - 2 \cdot c^* \cdot m \tag{2}$$

$$d_b = \frac{m \cdot z}{\cos\beta} \cdot \cos\alpha_t \tag{3}$$

$$\alpha_t = \tan^{-1}\left(\frac{\tan\alpha}{\cos\beta}\right) \tag{4}$$

$$s_n = m \cdot \left(\frac{\pi}{2} + 2 \cdot x \cdot \tan\alpha\right) \tag{5}$$

$$s_{bn} = z \cdot m \cdot \cos\alpha \cdot \left(\frac{s_n}{z \cdot m} + inv\alpha_t\right) \tag{6}$$

$$\beta_b = \tan^{-1}\left(\frac{d_b \cdot \tan\beta}{\frac{m \cdot z}{\cos\beta}}\right) \tag{7}$$

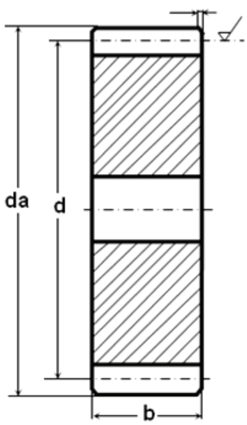
$$p_{bn} = m \cdot \pi \cdot \cos\alpha \tag{8}$$

Where:

z : number of teeth
m: normal module (mm)
x: addendum modification coefficient
β: helix angle at a reference diameter (°)
d_a: tip diameter (mm)
d_f: root diameter (mm)
a_w: centre distance (mm)
d_b: base diameter (mm)
β_b: base helix angle (°)
s_n : normal tooth thickness on reference cylinder (mm)
s_{bn} : normal base tooth thickness (mm)
p_{bn} : normal base pitch (mm)
α : pressure angle (°)
α_t : transverse pressure angle (°)
h_a* : factor of addendum
c* : factor of radial clearance

Moreover, standards (Norma NC 02-04-04, 1978; ISO Standard 1340, 1976; AGMA Standard 910-C90; 1990) with guidelines about the complete information to be given to the manufacturer in order to obtain the gear required give you an idea about the proper data to be placed on drawings of gears for general or special purposes. The mentioned information includes details of the gear body, the mounting design, facewidth, and fundamental gear data for manufacturing, inspection and reference. Usually, the gear data can be efficiently and consistently specified on the gear drawing in a standardized block format. Figure

1 shows the typical gear data block and information required on drawings for standard helical gears according to Cuban Standard NC 02-04-04:1998.



Normal module		m	
Number of teeth		z_1	
Helix angle at reference diameter		β	
Direction of helix angle, (LH or RH)			
Addendum modification coefficient		x	
Gear accuracy grade			
Basic rack tooth profile	Pressure angle	α	
	Factor of addendum	ha^*	
	Factor of tooth depth	hp^*	
	Factor of root fillet radius	ρ_f^*	
	Factor of radial clearance	c^*	
Data for control of corresponding flanks of gear teeth		s	
		h	
Reference diameter		d	
Mating gear	Number of teeth	z_2	
	Drawing number		

Figure 1. Typical data for gear drawings to be given by the gear designer for the gear manufacturer, according to NC 02-04-04: 78

3.0 INITIAL DATA AND MEASUREMENTS

In the proposed procedure, to calculate the fundamental gear tooth data of an external parallel-axis cylindrical involute gear, it is necessary to know the following parameters:

- Number of teeth on pinion and gear (z_1, z_2)
- Tip diameters on pinion and gear (da_1, da_2) in mm
- Facewidth on pinion and gear (b_1, b_2) in mm
- Base tangent length spanned in k teeth on pinion and gear (Wk_1, Wk_2) in mm
- Number of teeth spanned for the base tangent length on pinion and gear (k_1, k_2)
- Tooth depth on pinion and gear (h_1, h_2) in mm
- Centre distance (a_w) in mm
- Helix angle at tip diameter (βa) in degree

Number of teeth (z): Special care should be had counting the quantity of teeth in the gears. It is a good practice to make some mark with chalk in the tooth where the count begins to assure that the number of teeth was correctly determined. An incorrect specification of the number of teeth on gears will be catastrophic in the next calculation.

<u>Tip diameters (*da*):</u> A conventional vernier caliper of suitable size can be use to determine the distance between the two outer extremities of external gear teeth in position diametrically opposed. The measure will always be more accurate in gears with even number of teeth, but it is also practically applicable in gears with odd quantity of teeth, always better in gears with large number of teeth.

<u>Facewidth (*b*):</u> It is the width over the toothed part of a gear, measured along a generator of the reference cylinder. The measurement can be made using a vernier caliper, although it can be enough a simple rule with precision of millimetres.

<u>Base tangent length (*Wk*):</u> The measurement is made over a group of teeth using a conventional vernier calliper or plate micrometer. For a good results is required that the controlled flanks are perfectly clean and without appreciable wear. Moreover, the calliper jaws must penetrate sufficiently into two tooth spaces to make tangent contact with the tooth surfaces without interfering with the teeth adjoining the span measurement. Thus, the measurement of the distance between two parallel planes tangent to the outer flanks of a number of consecutive teeth, along a line tangent to the base cylinder, is taken. In case of not considering space between non-working flanks of the mating gears when the working flanks are in contact (zero backlash), the distance measured is equal to the normal thickness of one tooth at the base cylinder sbn plus the product of the number of teeth spanned less one (k -1) and the normal base pith pbn , see Equation (9). Suffixes k1 (for pinion) and k2 (for gear) after the letter W specifies the number of teeth between the flanks measured. Figures 2 and 3 illustrate the span measurement applied to spur and helical gears.

$$W_{tk} = s_{bn} + p_{bn} \cdot (k-1) \tag{9}$$

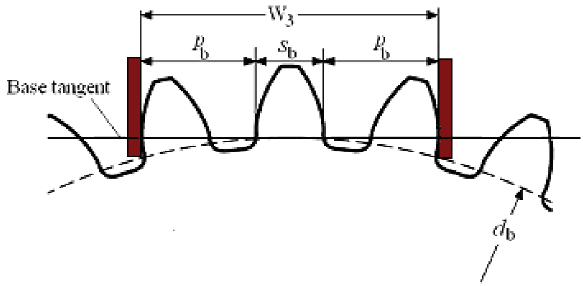


Figure 2. Measurement of base tangent length over 3 teeth in spur gear

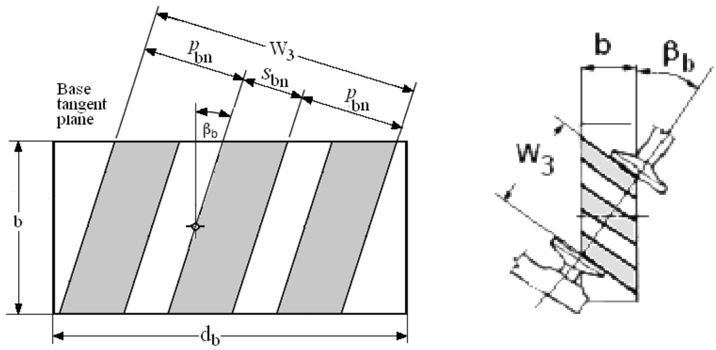


Figure 3. Measurement of base tangent length over 3 teeth in helical gear (top view)

On external parallel-axis cylindrical involute gear, the actual base tangent lengths (W_{k1} and W_{k2}) are less than the theoretical dimensions for zero backlash by the necessary amount of the normal backlash allowance, but this fact doesn't affect the practical results because standard values of gear backlash (ISO/TR 10064-2, 1996) are relatively small (not bigger than 3 or 7 % of module) for industrial drives with typical commercial manufacturing tolerances.

In gear with profile or helix modifications, the span measurement should be carried out on the un-modified part of the tooth flank. Moreover, in some case, span measurement cannot be applied when a combination of high helix angle and narrow facewidth prevent the caliper from spanning a sufficient number of teeth, see Equation (10). In this situation should be considered other alternative procedures to determine the unknown gear geometry using conventional measurement tools (Regalado, 2000) or exhaustive search method with a trial and error procedure to obtain an approximate geometry of the analyzed gears.

$$b_{\min} = 1{,}015 \cdot W_k \cdot \sin \beta_b \tag{10}$$

Where:

bmin: minimum value for facewidth in mm. There is an additional value of 1,5% to make an stable span measurement.

Number of teeth spanned for the base tangent length (k): In case of gears with tooth data specified, the number of teeth spanned for the base tangent length can be calculated (Maag, 1990), but for gears with unknown geometry, the number of teeth between the measuring surfaces can be established so that the points of contact with vernier caliper or plate micrometer are roughly at mid tooth height. The number of teeth to be spanned will be larger for gears with larger numbers of teeth and for gears with higher helix angle. Recommendations on Table 1, based on author's experiences and calculation of the base tangent length, can be used as guideline values of number of teeth for span measurement. For more detailed information about values of the number of teeth spanned for the base tangent length from the helix angle, the number of teeth, pressure angle and the addendum modification coefficient can be obtained in MAAG Gear Book.

Table 1. Guideline for the number of teeth spanned for the base tangent length

Helix angle at a tip diameter (β_a)	Number of teeth between the measuring surfaces so that the points of contact are roughly at mid tooth height (k).											
	2	3	4	5	6	7	8	9	10	11	12	13
	Number of teeth (z)											
0°	11	18	28	36	44	55	65	75	85	95	100	110
	18	28	36	44	55	65	75	85	95	100	110	120
20°	10	16	24	30	42	48	55	65	75	85	95	100
	16	24	30	42	48	55	65	75	85	95	100	110
40°	6	9	14	18	24	28	32	36	42	46	50	54
	9	14	18	24	28	32	36	42	46	50	54	60

Tooth depth (h): This magnitude is usually specified as the radial distance between the tip and root diameters. Tooth depth may be measured by means of a gear tooth vernier caliper or in tooth space using a simple vernier caliper with blade for depth measurements. The calliper blade must penetrate sufficiently and to make contact with the surface at the bottom of a tooth space without interfering with adjacent teeth flanks.

Centre distance (a_w): Involute gears can operate correctly with small change of centre distance according with the proper tolerances for deviations, but assembled gears with incorrect operating centre distance will not operating properly, for that reason, the centre distance should be determined with a good precision. This magnitude is accepted as the shortest distance between the axes of a gear pair and this is also the distance between the axes of shafts that are carrying the gears.

A common method to determine the gear centre distance is the measurement in parallel planes of the center holes distance located in their functional shafts, but taking into account the accuracy of cylindrical bearing seatings on shafts and in housing bores, a more satisfactory method is consider the nominal centre distance as the sum of the housing bores radii (or outer radii of bearings) plus the distance between them. Figure 4 and Equation (11) show this idea. Usually, speed reducers and enclosed gear units boxes have specified the nominal centre distance based on series of preferred numbers (IS0 Standard 3, 1973) and checking it may provide clues to nominal value of the centre distance.

$$a_w = R_1 + R_2 + T \tag{11}$$

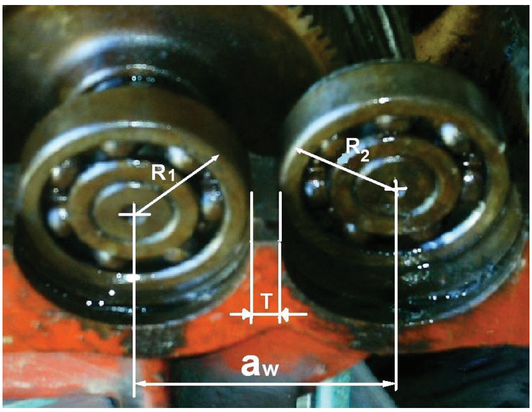


Figure 4. Parameters for calculation of centre distance (aw) by means of center holes distance or bearing housing bores radii (R1 + R2) plus the distance (T) between them

Helix angle at tip diameter (βa): For spur gears $\beta = \beta a = 0°$, because the helix is a straight line parallel to its rotating axis, but in case of helical gears the measuring of the helix angle at reference diameter is one of the most difficult of specifying and should be done with an special helix angle tester. When a helix angle measuring is not possible with these special equipments, the helix angle at reference diameter can get by a simple method based in the approximate measured of the helix angle at tip diameter (βa) with results not too exact, but practically acceptable. For this, it is necessary apply a marking compound to the tip surface of external gear teeth and roll the helical gear in straight line on a white paper to collect their generated trace (see Figure 5).

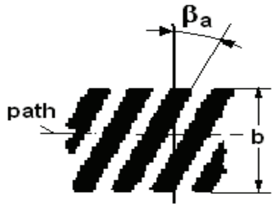


Figure 5. Measuring of outer helix angle βa by their generated trace

4.0 DETERMINATION OF THE UNKNOWN GEAR GEOMETRY

The output results of the unknown gear have strong relation with the measured values and depending of uncertainty of the measuring and including all manufacturing errors, wear and deformation on flanks in the gear itself. It is important understand this concept because modules, pressure angles, helix angles, addendum modification coefficient and other gear geometry features are given at calculated values and they are not necessarily the values used in the initial manufacturing of the gears, but they are very useful as reference to establish the fundamental parameters for reproduction of new gears or evaluation of the load capacity of gears.

With the initial data and measurements above mentioned, fundamental gear geometry parameters according to ISO standards can be obtained applying the following calculations.

Normal module (*m*)

The module *m* in the normal section of the gear is the same module *m* of the standard basic rack tooth profile (IS0 Standard 53, 1998) and is defined as the quotient of the pitch *"p"* (distance measured over the reference circle from a point on one tooth to the corresponding point on the adjacent tooth of the gear), expressed in millimetres, to the number π.

$$m = \frac{p}{m} \tag{12}$$

The module is a commonly referenced gear parameter in the ISO gear system and very important to defined the size of gear tooth. The module cannot be measured directly from a gear; yet, it is a common referenced value. Tooling for commercially available cylindrical gears are stocked in standardized modules (ISO Standard 54, 1996) (ANSI/ AGMA 1102-A03, 2003). Generally, when gear generation has ended a perfect engagement between gear and its generating hob occurs.

Thus, the normal module in the unknown gear geometry may be determined by a simple search of gear generating hob with known module which has a perfect mating with the analyzed gear, but this procedure requires of a complete set of generating hob to give solutions and it is not economically desirable, especially when the measurement has to be done in the field. Moreover, the normal module could be determined using a more practical procedure based on the difference between values of base tangent lengths over a consecutive number of teeth spanned and their relations with the normal base pitch. Once the base tangent lengths have been measured, the value for reference of the normal module may be calculated applying Equations (13) and (14) for pinion and gear respectively. Since the values m1 and m2 need not be exactly precise can be taken for calculation propose a value of $\alpha \cong 20°$.

$$m_1 = \frac{W_{k1} - W_{k1-1}}{\pi \cdot \cos\alpha} \tag{13}$$

$$m_2 = \frac{W_{k2} - W_{k2-1}}{\pi \cdot \cos\alpha} \tag{14}$$

Although mating gears can have different base tangent lengths and number of teeth, mating gears must have the same module and pressure angle, for that reason the correct normal module for gear *m* should be established equal to the nearest standardized module to the values m_1 and m_2. Table 2 can be used as guideline for values of standardized normal modules.

Table 2. Standardized normal modules of cylindrical gears for general and heavy engineering

Series	I	1	1,25	1,5	2	2,5	3	4	5	6
	II	1,125	1,375	1,75	2,25	2,75	3,5	4,5	5,5	(6,5)
Series	I	8	10	12	16	20	25	32	40	50
	II	7	9	11	14	18	22	28	36	45

NOTE: Preference should be given to the use of the normal modules as given in series I. The module 6,5 of series II should be avoided. These normal modules are not necessary applicable to gears used in the automotive field.

Helix angle at reference diameter (β).

As it is known, in spur gears the helix angle at reference diameter is β= 0°. In case of helical gears the helix angle at reference diameter can be calculated based in the measured of the helix angle at tip diameter (βa) as follows:

$$\beta = sen^{-1}\left(\frac{m \cdot z \cdot \tan\beta_a}{d_a}\right) \tag{15}$$

Nominal pressure angle (α).

It is an important characteristic of the standard basic rack tooth profile for cylindrical involute gears cutting by generating tool and constitutes a geometrical reference for involute gears in order to fix the sizes and profiles of their teeth. In general, gears are generating with a cutter normal profile angle chosen from the range between 14.5° and 25°. Standard values for nominal pressure angle are 14.5°, 17.5°, 20°, 22.5°, and 25°. Some gear manufacturers use non-standard cutter profile angles to accomplish specific design goals, in these case this method of reverse engineering can give some idea for recreating other new gear with standardized values of pressure angle.

Taking into account the sum of theoretical base tangent lengths of both toothed wheels ($\sum w_{tk} = W_{tk1} + W_{tk2}$) the nominal pressure angle can be estimated. By means of mathematical processing of the Equations (6), (8), (9) and (16) for pinion and gear is possible the determination of Equation (17). In particular, Equation (17) is relevant because the numerical values obtained are derived directly from the basic gear data specified previously and can be used as important factor in the decision making task.

$$x_\Sigma = x_1 + x_2 = \frac{(inv\alpha_{wt} - inv\alpha_t)}{2 \cdot \tan\alpha} \cdot (z_1 + z_2) \tag{16}$$

$$\Sigma w_{tk} = w_{tk1} + w_{tk2} = \cdot[\pi \cdot (k_1 + k_2 - 1) + (z_1 + z_2) \cdot inv\alpha_{tw}] \cdot m \cdot \cos\alpha \tag{17}$$

With

$$\alpha_t = \tan^{-1}\left(\frac{\tan\alpha}{\cos\beta}\right)$$

$$\alpha_{tw} = \cos^{-1}\left(\frac{m \cdot (z_1 + z_2) \cdot \cos\alpha_t}{2 \cdot a_w \cdot \cos\beta}\right)$$

$$inv\alpha_{wt} = \tan(\alpha_{wt}) - \alpha_{wt}$$

Where:

$\sum w_{tk}$: sum of theoretical base tangent lengths of mating pinion and gear.

α_{wt} : pressure angle at the pitch cylinder.

α_t : transverse pressure angle

To determine the nominal pressure angle in the unknown gear should be compared the sum of the theoretical base tangent lengths ($\sum W_{tk} = W_{tk1} + W_{tk2}$) with the result of the sum of the measured base tangent lengths ($\sum W_k = W_{k1} + W_{k2}$). Thus the nominal pressure angle α must be estimated equal to the nearest standard value of pressure angle with smaller difference between the sum of the theoretical ($\sum W_{tk}$) and measured ($\sum W_k$) base tangent lengths of both gears. The starting value in the search should be 20°, since the majority of cutting tools use that angle conforming to world-wide acceptance. Smaller pressure angles can be analyzed for case of gears with higher transverse contact ratios when lower noise levels are desirable, in this circumstances these gears usually have high numbers of teeth and lightly loaded. Higher pressure angles are sometimes preferred for gears with lower numbers of teeth and heavily loaded when tooth bending strength is required. Table 3 shows a sample of how to determine a nominal pressure angle.

Table 3. Sample of the procedure to determine the standardized pressure angle by means of difference between the sum of the theoretical ($\sum W_{tk}$) and measured ($\sum W_k$) base tangent lengths of both gears

Basic gear data

Pinion (1):

- Number of teeth $z_1 = 16$
- Number of teeth between measured flanks $k_1 = 2$
- Actual base tangent length (average) $w_2 = 13{,}88$ mm

Wheel (2):

- Number of teeth $z_2 = 83$
- Number of teeth between measured flanks $k_2 = 10$
- Actual base tangent length (average) $w_{10} = 87{,}48$ mm

Gear:

- $m = 3$ mm
- $\beta = 8{,}11°$
- $a_w = 150$ mm

Nominal pressure angle (α)	14,5º	17,5º	
Pressure angle at the pitch cylinder (α_{tw})	14,64º	17,66º	
inv α_{tw}	0,00571	0,01015	
Sum of the theoretical base tangent lengths ($\Sigma W_{tk} = W_{tk1} + W_{tk2}$); mm	102,01	101,75	
Sum of the actual base tangent lengths ($\Sigma W_k = W_{k1} + W_{k2}$); mm			101,36
Difference between theoretical (ΣW_{tk}) and measured (ΣW_k); mm	0,65	0,39	
	Estimated value of standardized pressure angle α = 20º		

Addendum modification coefficient (*x1, x2*)

The profile shift is the amount that is added to, or subtracted from, the gear teeth addendum to enhance the operational performance of the gear mating or meet fixed design criteria. For specialists involved with gear design based on ISO standards, it's very familiar that the datum line of the basic rack profile need not necessary be tangent to the reference diameter on gear, thus the tooth profile and his shape can be modified by shifting the datum line from the tangential position (González Rey, G. et al, 2006). The main parameter to evaluate the addendum modification is the addendum modification coefficient *x*, also know by American as profile shift factor or rack shift coefficient.

The addendum modification coefficients for pinion (*x1*) and gear (*x2*) can be estimated by Equations (18) and (19) obtained by consideration of normal backlash and mathematical processing of the Equations (6), (8), (9) and (15)

$$x_1 = \frac{1}{2 \cdot \tan\alpha} \cdot \left[\frac{W_{k1} + j_{bn}}{m \cdot \cos\alpha} - \pi \cdot (k_1 - 0{,}5) - z_1 \cdot inv\alpha_t \right] \tag{18}$$

$$x_2 = \frac{(inv\alpha_{wt} - inv\alpha_t)}{2 \cdot \tan\alpha} \cdot (z_1 + z_2) - x_1 \tag{19}$$

Where: j_{bn} = Normal backlash (mm).

Normal backlash is the shortest distance between non-working flanks of two gears when the working flanks are in contact. Some backlash should be present in all gear meshes. It is required to assure that the non-driving flanks of the teeth do not make contact. Backlash in a given mesh varies during operation as a result of changes in speed, temperature and load. The amount of backlash required depends on the size of the gears, their accuracy, mounting and the application. For purpose of this procedure, normal backlash is preferable measured with feeler gauges when gears are mounted in the housing under static conditions. When normal backlash can not be measured can be used Table 4 as guideline of values of minimum backlash (ISO/TR 10064-2, 1996) recommended for industrial drives with ferrous gears in ferrous housings, working at pitchline speeds less than 15 m/s, with typical commercial manufacturing tolerances for housings, shafts and bearings.

Table 4. Recommended values (in mm) for minimum backlash j_{bn}

Normal module (m); mm	Centre distance (a_w); mm				
	50	100	200	400	800
1,5	0,09	0,11			
2	0,10	0,12	0,15		
3	0,12	0,14	0,17	0,24	
5		0,18	0,21	0,28	
8		0,24	0,27	0,34	0,47
12			0,35	0,42	0,55

Equations (20) and (21), derived from the Equation (5), give a possible cross-check for the estimated values of addendum modification coefficients if the normal tooth thicknesses on reference cylinder for pinion and gear (s_{n1} and s_{n2}) are known.

$$x_1 = \frac{\frac{s_{n1}}{m} - \frac{\pi}{2}}{2 \cdot \tan \alpha} \tag{20}$$

$$x_2 = \frac{\frac{s_{n2}}{m} - \frac{\pi}{2}}{2 \cdot \tan \alpha} \tag{21}$$

<u>Factor of radial clearance (*c**) and factor of addendum (*ha**)</u>

Shape and geometrical parameters of the basic rack tooth profile for involute gears are setting by special standards (see Table 5) in corresponding with the rack shaped tool (such as hobs or rack type cutters) used in the cutting of gear by means of generation methods. The dimensions of the standard basic rack tooth profile give information about standardized values of radial clearance and addendum as a multiple of the normal module.

Table 5. Some standard values of basic rack tooth profile parameters.

α	h_a^*	c^*	ρ_F^*	Standard
20,0°	1,00	0,25	0,380	ISO 53:1998
20,0°	1,00	0,25	0,300	
20,0°	1,00	0,25	0,250	
20,0°	1,00	0,40	0,39	
20,0°	1,00	0,25	0,300	AGMA 201.02-68
20,0°	1,00	0,25	0,350	
20,0	0,8	0,20	0,3	
25,0°	1,00	0,25	0,300	
25,0°	1,00	0,25	0,350	
14,5°	1,00	0,157	-	
20,0°	1,00	0,25	0,375	JIS B 1701-72
20,0°	1,00	0,25	0,400	GOST 13755-68

The factor of radial clearance is the distance, along the line of centres, between the root surface of a gear and the tip surface of its mating gear given in relation to normal module. Radial clearance is the same between the root surface and the tip surface for pinion and gear with the same tooth depth (see Figure 6).

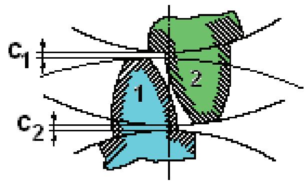


Figure 6. Identification of radial clearances between the teeth of gear meshes

Equations (22) and (23) can be used to determine the factor of radial clearances. For purpose of this procedure, radial clearances are preferable measured with gauges when gears are mounted in the housing under static conditions.

$$c^*_1 = \frac{c_1}{m} = \frac{\left[a_w - 0{,}5 \cdot (da_1 + da_2) + h_2\right]}{m} \tag{22}$$

$$c^*_2 = \frac{c_2}{m} = \frac{\left[a_w - 0{,}5 \cdot (da_1 + da_2) + h_1\right]}{m} \tag{23}$$

Equations (24) and (25), derived directly from the basic gear are given to estimate values of factor of addendum.

$$h^*_{a1} = \frac{da_2 - 2 \cdot a_w + \left({m \cdot z_1}/{\cos\beta} \right)}{2 \cdot m} + x_1 \tag{24}$$

$$h^*_{a2} = \frac{da_1 - 2 \cdot a_w + \left({m \cdot z_2}/{\cos\beta} \right)}{2 \cdot m} + x_2 \tag{25}$$

Since the majority of cutting tools use values of ha* = 1 and c* = 0,25, conforming to world-wide acceptance, these values should be analysed firstly in the searching. It is possible to found other non-standard cutter to accomplish specific purpose as ha* = 0,75 for stub gears or ha* = 1,25 for gears with deep teeth. In case of non-standard system of basic rack tooth profile, Equations (22) to (25) can give some idea for recreating other new gear with standardized values.

5.0 CONCLUSIONS

The theory of the involute surface of the flank of a cylindrical gear can give information about basic gear tooth data needed to determine the unknown gear geometry. Based in the mentioned theory, a procedure of reverse engineering to determine the basic geometry of external parallel-axis cylindrical involute gears has been presented. The proposed method can be used as an alternative procedure to determine the unknown gear geometry using conventional measurement tools.

It is important to highlight that all results, more or less accurate, represent the estimated values of the gear mating, depending on the uncertainty of the measurement and including all manufacturing errors in the gear itself. This is an important concept because modules, pressure angles, helix angles, addendum modification coefficients and other gear geometry features determined using this method are given at estimated values and they are not necessarily the values used in the initial manufacturing of the gears, but they are very useful as reference to establish the fundamental parameters for the evaluation of the load capacity of cylindrical gear or the reproduction of a new gear pair.

The method, based on author's experiences in the analysis, recovery and conversion of helical and spur gears according to ISO, AGMA and non-standard gear systems, proposes a practical method with results not too exact, but practically acceptable, to obtain by calculating and conventional measurement tools the fundamental parameters needed for the reproduction of a new cylindrical gears according to ISO standards.

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