Design and Analysis of Centrally Suspended Cage-less Limited Slip Differential

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Abstract

Limited slip differential (LSD) plays an important role in maintaining speeds and in torque distribution between the wheels of the automobile even in case of variable traction. In the present project work, limited slip differential is considered and the objective of the project is to find out tangential, axial and radial forces involved in meshing of the differential gears theoretically. Finite Element Analysis (FEA) is carried out on Final, Crown, Side and Ring gears made of 20MnCr5 material and running at a speed of 4000rpm and torque of 122N-m. From the analysis, it is found that the forces and stresses obtained are below the allowable stress of the material considered in designing gears of the differential. Maximum von-Mises stress is for Ring gear and Minimum for Side gears.

Keywords: Centrally Suspended Cage-less Limited Slip Differential, von-Mises stresses, Beam and Wear strength, Factor of safety.

1.0 INTRODUCTION

Limited-slip differential (LSD) is used when a vehicle equipped with a standard differential spins a tire but the opposite wheel does not receive enough torque to move the vehicle. To solve this problem, most manufacturers use differentials that direct more power to the side gear attached to the spinning axle. Many differentials do this by forcing the side gear against the revolving case. This bypasses differential action, allowing the case to drive the side gear directly. A limited-slip differential distributes torque to both wheels equally or unequally, allowing the wheels to turn at the same or at different speeds. The only means of having the standard differential apply different amounts of torque to each axle is to have the case drive the side gear directly, bypassing the pinion gears. One means of accomplishing this is to literally “push” the side gear out of mesh with the pinion gears against the rotating case. There are many types of LSD based on different designs but in present paper, we are using a Clutch pack/ pressure disk type LSD. Here, apart from the basic components of LSD, it has a series of friction and steel plates packed between the side gear and the casing. Friction discs have internal teeth and they are locked with the splines of the side gear. So the friction discs and the side gear will always move together. Steels plates have external tabs and are made to fit in the case groove. So they can rotate with the case. If any of the clutch pack assembly is

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well pressed, the frictional force within them will make it move as a single solid unit. Since steel plates are locked with the case and friction discs with the side gear, in a well pressed clutch pack casing and the clutch pack will move together or motion from the casing is directly passed to the corresponding axle. Space between the side gears is fitted with a pre-load spring. Pre load spring will always give a thrust force and will press clutch pack together (Figure.1).

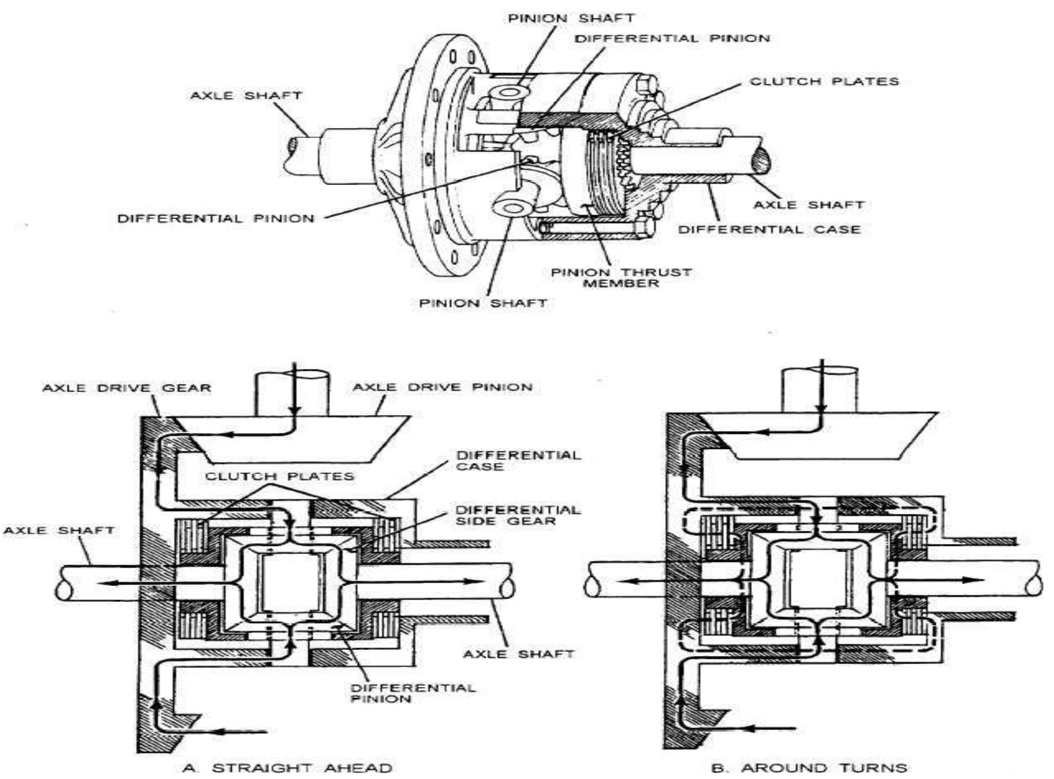


Fig 1: Clutch pack/ Pressure disk Limited Slip Differential
A. while moving straight ahead, B. while taking turns[8]

2.0 LITERATURE REVIEW

Prof. A. S. Todkar, R. S. Kapare [1] designed a differential locking system that can be engaged or disengaged either manually or automatically. They concluded that automatic engagement of the differential when the loss of traction condition is encountered thereby validating the function of the automatic mode of the differential locking system and manual override using push button system for semi-automatic mode of differential locking.Daniel Das.A et al. [2] carried out mechanically design and analysis on assembly of gears in a differential gearbox when they transmit power at different speeds and observed the structural analysis results, using Aluminum alloy the stress values are within the permissible stress value. So using Aluminum Alloy is safe with decreased vibrations and increased mechanical efficiency.Nilton Raposo et al. [3] designed a differential unit and analyze using FEM methods and illustrated the entire design methodology into the differential and driveline assembly.Suraj Aru et al. [4] designed a cage-less differential and analyze its use on an All-

Terrain Vehicle (ATV)and illustrated the entire design methodology of a cage less differential.

Muhammad Safuan Bi Md Salleh[5] carried out mechanically design and analysis on assembly of gears in gearbox and concluded that the cast iron material is replaced to Aluminum Alloy for reducing weight. N. Siva Teja[6] performed mechanical design of differential gear box and analysis of gears in gear box and found that both grey cast iron and aluminum alloy are preferable for performing the application of differential gearbox in automobiles. But, when it comes to weight for light utility vehicles Aluminum Alloy is preferred.

T.Loknadhet al.[7] designed of crown wheel gear used in gear boxand did structural analysis to verify the best material (steel and Aluminum Alloys) for the crown wheel gear in the gear box at higher speed by analyzing static and dynamic conditions.Concluded design of crown wheel gear to be safe under the above operating loading conditions. But the steel material model factor of safety (2.9) is better than the aluminum material model FOS (1.64). Hence we can conclude that the steel material design is better than the aluminum material design.

Gabriele Barbaraci et al. [8] designed a MR sports car differential, with the aim of the total mass Reduction and torquemaintenance and concluded that Possibilities offered by MR fluids for the implementation of a limited slip differential are studied in this paper.Mayank Bansal et al. [9] studied the structural and dynamic behavior of the gears in the differential gearbox assembly made up of the different composite material as compared to the conventional metallic materials and concluded that the composites material can successfully replace the steel gear for the gearbox application. The FEM based static analysis at different loading condition shows that the total deformation and stress induced in the material were less for the composite as compared to others metals considered. N.Vijayababu et al. [10] conducted analysis to verify the best material for the gears in the gear box at higher speeds by analyzing stress, displacement and also by considering weight reduction and observed that structural analysis results using Aluminum alloy the stress values are within the permissible stress value. So using Aluminum Alloy is safe for differential gear.

Richard A.James [11] designed a lightweight aluminum differential housing to replace the cast-iron housing used in the Torsen T-1 and the cast-iron housing was evaluated and redesigned as a two-piece aluminum unit. S.H.Gawande et al. [12] performed mechanical design of crown wheel and pinion in differential gear box of MFWD (FWA) Axle and detailed manual and computer aided designing of crown gear and pinion is carried out and concluded the design to be safe. G.Srikanth Reddy [13] designed and analyzed gears assembly in differential gear box and concluded that by theoretical comparison between steel and Ni-Cr steel, Ni-Cr steel is better in differential gear box manufacturing because of its high strength, Though the cost of Ni-Cr is high, it has long life compared to Steel.

Shashank Pandey et al. [14] analyzed the differential gears assembly and its housing for the vibrational effect on a system in which the life of the gears is determined within different frequency range and in this research concluded that 3D deformable body model of differential gearbox and its housing was developed through SOLIDWORKS. The results obtained were then compared with the AGMA theoretical stress values. The results are in good congruence with the theoretical values, which suggest that the model designed is correct. Amir Khan [15] et al. obtained performance parameters of modified automobile central differential and to compare with Transfer gear box and using the FEA output, a design was created that met all the stipulated functional requirements and possessed a reasonable factor of safety while still saving significant mass and rotational inertia as compared to the old differential or transfer gearbox.

From the above literature, it is understood that earlier researches carried out works on analysis related to Von-Mises stresses, deformation, vibrational effects, harmonic effect, deflection, maximum contact stresses, fatigue & static analysis, torque distribution and replaced conventional materials of the differential with new materials for better results. The present project work comprises design of cage-less centrally suspended limited slip differential using Solid works 2017 and calculating the forces involved in limited slip differential. Carried out Static structural analysis to find Von-Moisses stresses and deformation of all differential gears by replacing differential gear materials to 20MnCr5 using ANSYS 14.5.

3.0 DESIGN CALCULATIONS

The following notations are followed in the design calculations:

Number of teeth on pinion	- Z_p
Number of teeth on gear	- Z_g
Pitch angle of pinion	- γ
Module	- m
Diameter of pinion	- D_p
Diameter of gear	- D_g
Pressure angle	- α
Face width	- b
Mean radius	- R_m
Tangential load	- P_t
Radial load	- P_r
Axial/Thrust load	- P_a
Torque	- M_t
Pitch of gear	- Γ
Formative spur gear tooth	-Z_p’
Ultimate strength	-σ_u
Allowable bending stress	-σ_b
Working stress	-σ_w
Beam strength	-S_b
Lewis form factor	-Y
Cone distance	-A_o
Wear strength	-S_w
Ratio factor	-Q
Material constant	-K
Brinell hardness number	-BHN
Sum of errors between Two meshing gear teeth-e	

Deformation factor -C
Effective load -P_{eff}
Dynamic load -P_d
Speed of pinion -N_p
Factor of safety
against bending failure -FS_b
Factor of safety
against pitting failure -FS_w
Factor of safety -Fs

To design differential gears, the following flow of calculations is adopted

$\tan\gamma = Z_p/Z_g$
Module (m) = D_p/Z_p
Mean radius of Final drive gear (R_m)= $[D_p/2 - b\sin\gamma/2]$
Tangential load (P_t) = M_t/R_m
Radial load (P_r) =$P_t.\tan\alpha.\cos\gamma$
Axial (or) Thrust load (P_a) = $P_t.\tan\alpha.\sin\gamma$
For gear pitch angle (Γ) = 90-γ
Mean Radius of Ring gear (R_m) = $[D_p/2 - b\sin\gamma/2]$
Torque on Ring gear (M_t) = $P_t \times R_m$

To calculateBeam and Wear strength

Allowable bending stress (σ_b) = $\sigma_u/3$
Pitch angle (γ)=$\tan^{-1}(40/60)$
Formative spur gear tooth (Z_p')=$Z_p/\cos\gamma$
Diameter of pinion (D_p) =$m \times Z_p$
Diameter of gear (D_g) =$m \times Z_g$
Cone distance (A_o) =Sqrt $[(D_p/2)^2+ (D_g/2)^2]$
Beam strength (Sb) =$m \times b \times \sigma_b \times Y\ [1-(b/A_o)]$
Ratio factor (Q) = $[(2 \times Z_g)/ (Z_g+Z_p\tan\gamma)]$
Material constant (K) =$0.16\times (BHN/100)^2$
Wear strength (Sw) = $(0.75 \times b \times Q \times D_p \times K)/\cos\gamma$
Velocity (v) = $(\pi \times D_p \times N_p)/ (60\times10^3)$
Dynamic load (Pd) = $[21 \times v \times (\{C \times e \times b\} +P_t)]/ [(21 \times v) +\text{Sqrt}(\{C \times e \times b\} +P_t)]$
Effective load (P_{eff}) = $(C_s \times P_t) +P_d$
Factor of safety against bending failure (FS_b) = S_b/P_{eff}
Factor of safety against pitting failure (FS_w) = S_w/P_{eff}

To calculate factor of safety

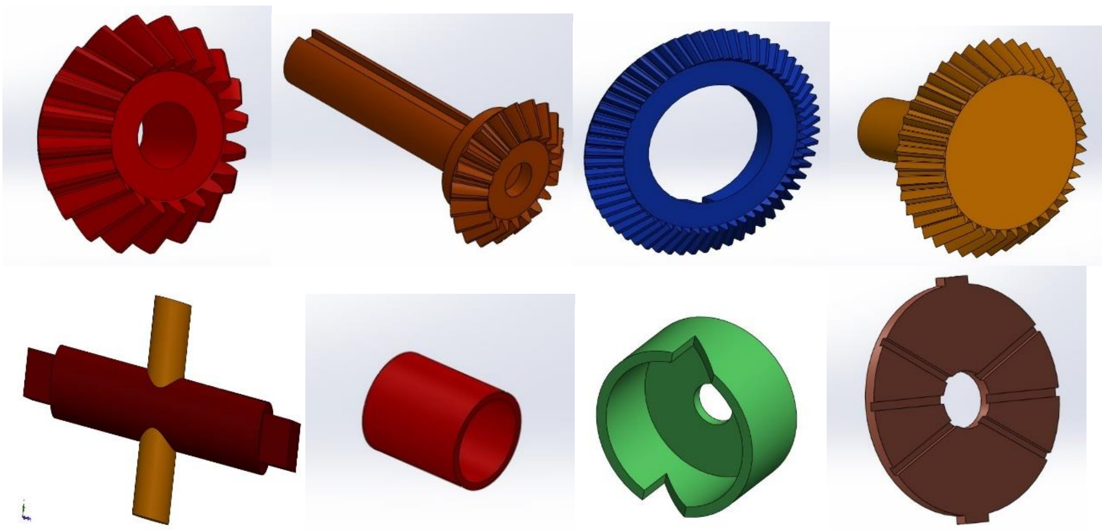
Working stress (σ_w) = $P_{eff/} \{m \times b \times Y \times [1-(b/A_o)]\}$
Factor of safety (Fs) =σ_u/ σ

4.0 MODELING

In present work, Centrally Suspended Cage-less limited slip differential consists of Crown gears, Side gears, a ring gear, a final drive gear, a cross pin, bushs, gear housings, pressure plates, friction plates and housing for complete differential.

Table 1: Details of components of Assembly

Sl.No	Component name	Quantity	Purpose
1	Crown gear	2	Mounted onto Cross pin over bush and transmits power from the Ring gear to Side gears.
2	Side gear	2	Directly connected to the wheels and transmit power from crown gears to the wheels
3	Ring gear	1	Connected to Cross pin connecting crown gears and transmits power from the final drive gear to the crown gears.
4	Final drive gear	1	Directly connected to the final drive and transmits power from the final drive to the ring gear.
5	Cross pin	1	Keeps the crown gears and side gears intact
6	Bush	2	Connects crown gears to the attachment.
7	Gear Housing	2	Houses all the crown gears and side gears and keeps all gears in position.
8	Pressure plate	-	Applies pressure on the friction plate engaging all plates in position and retains oil.
9	Friction plate	-	Acts as a friction lining and keeps plates engaged
10 & 11	Housing	1	Houses the entire assembly.



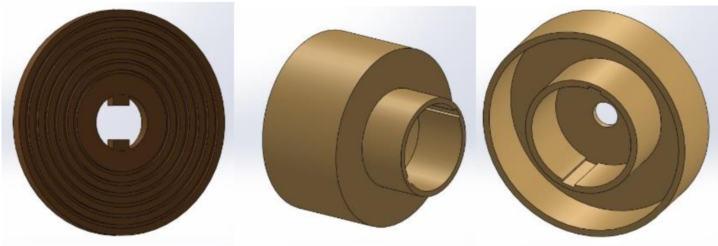


Fig 2: Components of Assembly (1. Crown gear, 2. Side gear, 3. Ring gear, 4. Final drive gear, 5. Cross pin, 6. Bush, 7. Gear housing, 8. Pressure plate, 9. Friction plate, 10. First half of Housing, 11. Second half of Housing)

All gears in the Assembly are designed using material 20MnCr5 and its properties are as follows:

Table 2: Properties of alloy used for differential gears

Sl.No	**Property**	**Value**
1.	Name of the alloy	20MnCr5
2.	Model type	Linear elastic isotropic
3.	Default Failure Criterion	Maximum Von-Mises
4.	Yield Strength	850 MPa
5.	Tensile Strength	1300 MPa
6.	Elastic Modulus	2E+5 MPa
7.	Poisson’s Ratio	0.285
8.	Mass Density	8000 Kg/m^3
9.	Shear Modulus	7.7821E+4 MPa
10.	Bulk modulus	1.5504E+5 MPa

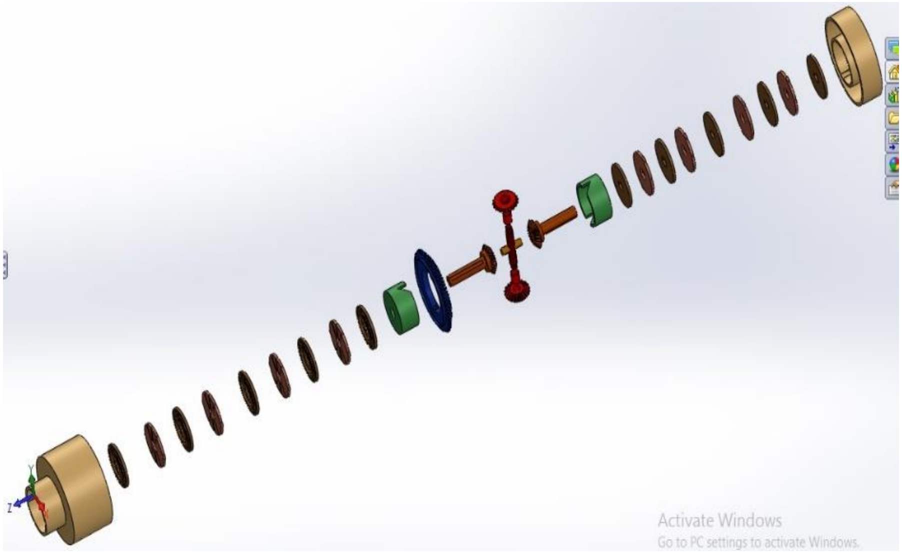


Fig 3: Exploded View of the Differential

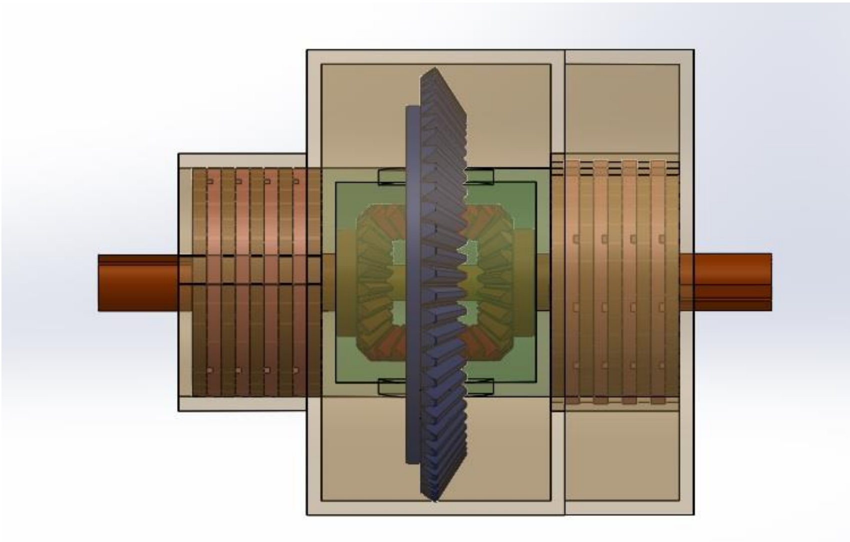


Fig 4: Assembled View of the Differential

5.0 STATIC STRUCTURAL ANALYSIS

A simple structural analysis is performed as the first step to see if components were structurally strong. If component fails due to loadings, then there is no need to continue any further analysis since the component is not strong enough to be used. The analysis of the various components of the differential was done in ANSYS 14.5 WORKBENCH for meshing as well as solving. Meshing of all the parts was done in ANSYS. The mesh is generated by using tetrahedron elements of 1 mm size. Mesh quality is further improved by using proximity and curvature function. This improves mesh density where curvature is small or edges are closed in proximity.

a) Static Structural analysis of a Crown gear of above differential assembly is as follows:

When designing a pair of bevel gears there are 3 forces which come into action. These are radial force (Pr), tangential force (Pt) and axial force (Pa). To analyze the bevel gears these three forces have been applied on gear's teeth. It is considered that there are 2 teeth in mesh with the bevel pinion. The constraints which were given are only physical constraints. The inner surface of the gear is fixed, where shaft is mounted and also the diagonally opposite surfaces of the slots which are made for fixing the pin.

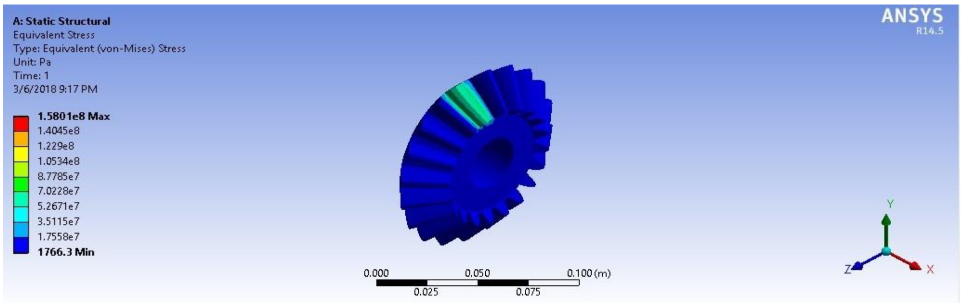


Fig 5: Stress analysis in ANSYS

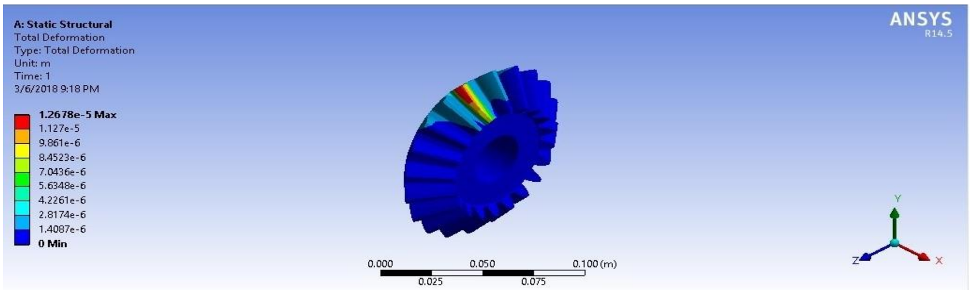


Fig 6: Maximum deformation in ANSYS

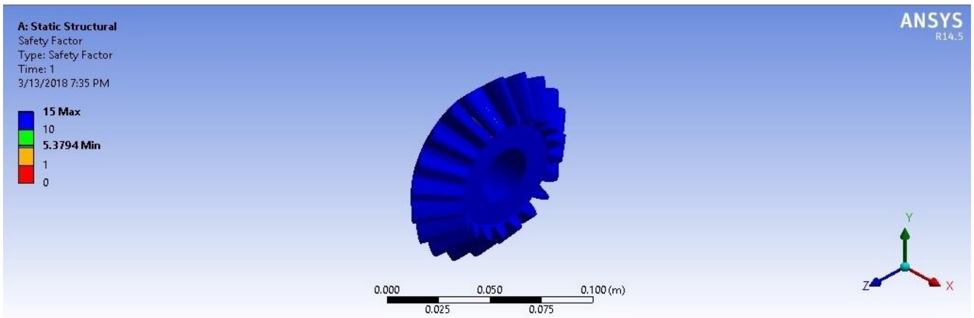


Fig 7: Safety factor in ANSYS

b) Static Structural analysis of a Side gear of above differential assembly is similar to that of Crown gears & is as follows:

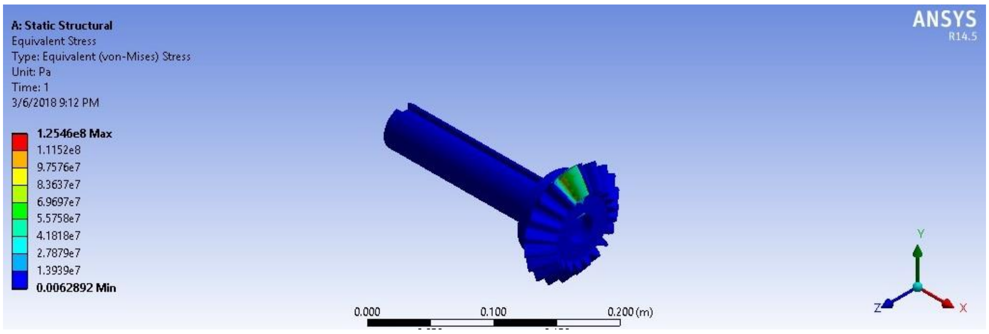


Fig 8: Stress analysis in ANSYS

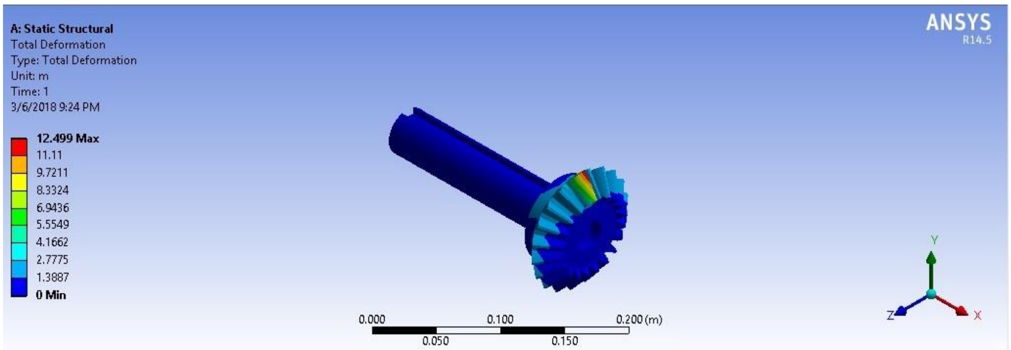


Fig 9: Maximum deformation in ANSYS

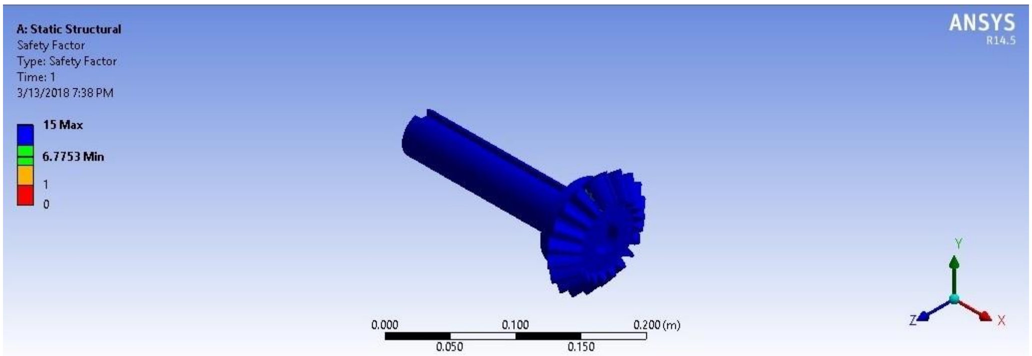


Fig 10: Safety factor in ANSYS

c) Static Structural analysis of a Ring gear of above differential assembly is as follows:

Gear teeth are subjected to both bending and wear. The section where it experiences the maximum stress is the root of the tooth. It is considered that, there are 3 teeth in contact with the mating gear. Tangential force (Pt) because of the torque which the gear is transmitting is applied on these three teeth. The constraints which were given are only physical constraints. The inner surface of the gear is fixed, where shaft is mounted and also the diagonally opposite surfaces of the slots which are made for fixing the pin.

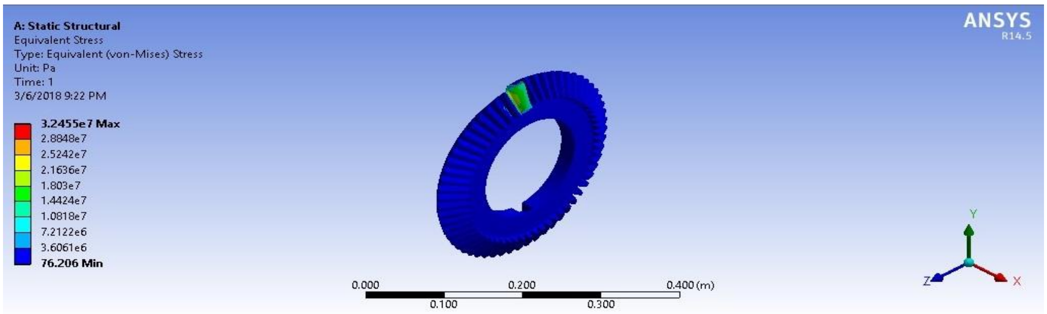


Fig 11: Stress analysis in ANSYS

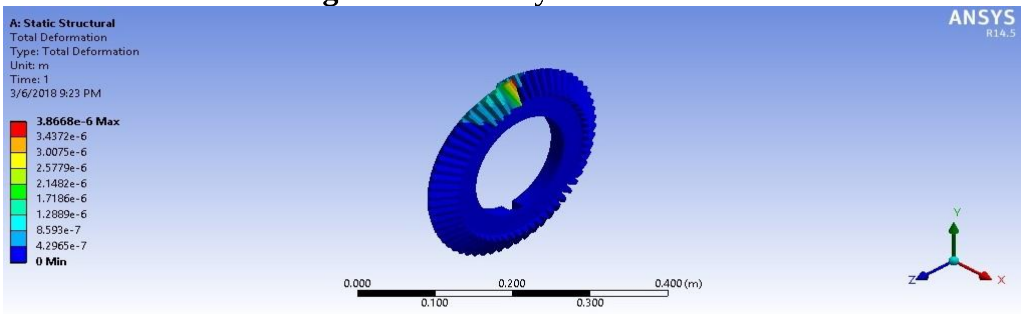


Fig 12: Maximum deformation in ANSYS

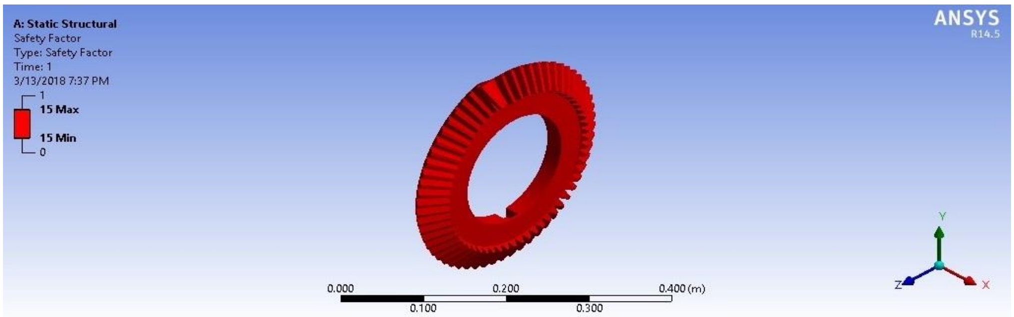


Fig 13: Safety factor in ANSYS

d) Static Structural analysis of a Final drive gear of above differential assembly is similar to that of Ring gear& is as follows:

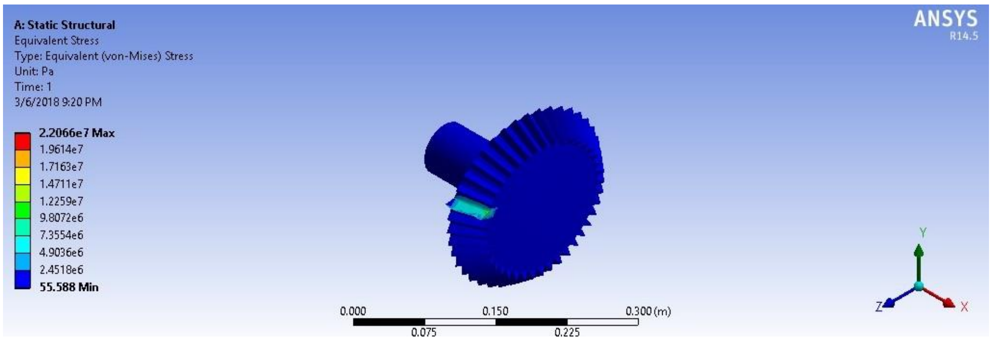


Fig 14: Stress analysis in ANSYS

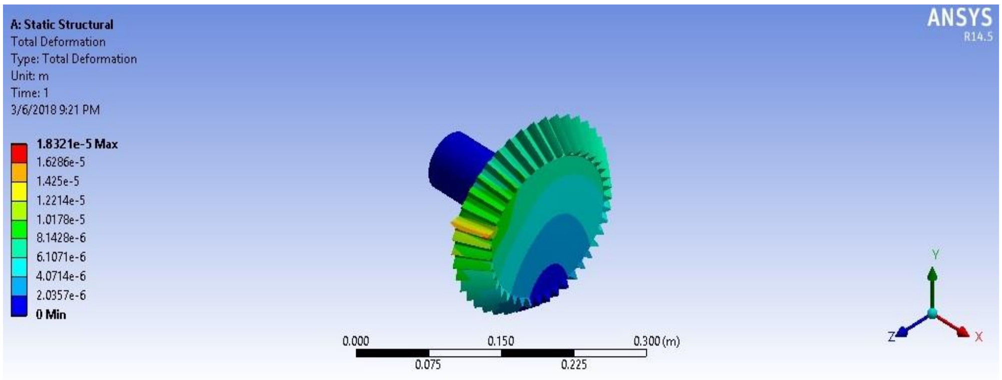


Fig 15: Maximum deformation in ANSYS

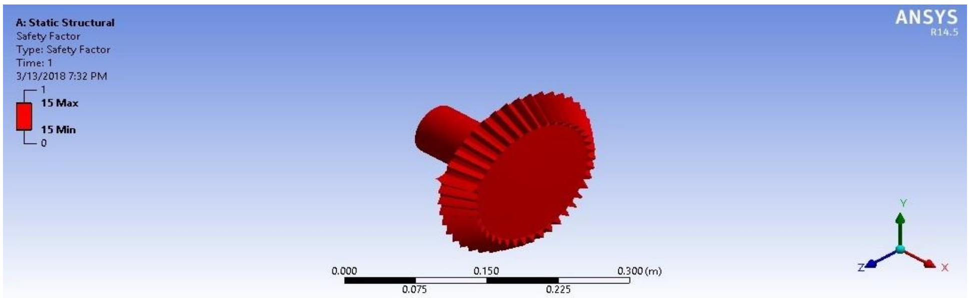


Fig 16: Safety factor in ANSYS

6.0 RESULTS & DISCUSSION

Design calculations for finding out tangential, radial, axial forces and factor of safety are calculated for differential gears of centrally suspended cage-less limited slip differential from the input torque given to the final drive gear. The results are tabulated as:

Table 3: Theoretically calculated forces

Sl.No	Gear name	Tangential Forces (N)	Radial Forces (N)	Axial/Thrust Forces (N)	Factor of safety
1	Final drive gear	1310.89	396.99	264.66	>10
2	Crown gears	2126.127	547.1929	547.1929	6.9
3	Side gears	2126.127	547.1929	547.1929	6.9
4	Ring gear	1310.89	264.66	396.99	>10

Beam and Wear strength of differential gears, safety factors against bending failure and pitting failure are calculated from Tangential forces acting on the differential gears and are tabulated in as:

Table 4: Beam and Wear strength of differential gears

Sl.No	Gear name	Beam strength (N)	Wear strength (N)	FS_b	FS_w	Effective load (N)
1	Final drive gear	20,178.32	35,954.53	3.46	6.1714	5825.962
2	Crown gears	11,204.9296	10,800	1.6048	1.5468	4855.786
3	Side gears	11,204.9296	10,800	1.6048	1.5468	4855.786
4	Ring gear	20,178.32	35,954.53	3.46	6.1714	5825.962

The beam strengths obtained above are greater than actual working Tangential load acting on the differential gears.The Beam and Wear strength of tooth of the differential gears is more than the effective load between the meshing teeth of differential gears. Hence, the design is safe.

Safety factors against bending and pitting failures are more than 1 and less than 7 to avoid unnecessary weight of Components which increases weight of the differential resulting in increase of the total weight of vehicle.

By conducting static structural analysis on the following differential gears, solutions obtained are maximum and minimum von-Mises stresses and factor of safety. The results obtained are tabulated in as:

Table 5: Von-Mises stresses

Sl.No	Gear name	Minimum stress (Pa)	Maximum stress (Pa)	Allowable stress (Pa)	Factor of safety
1	Final drive gear	55.588	2.2066e7	5.5e8	>10
2	Crown gears	1766.3	1.5801e8	5.5e8	5.3794
3	Side gears	0.0062892	1.2546e8	5.5e8	6.7753
4	Ring gear	76.257	3.2477e7	5.5e8	>10

Safety factors obtained here are close to those obtained in theoretical calculations. It is observed that Maximum stress on all differential gears is less than the allowable stress. So, the design is safe.

The displacements obtained after deformation in static structural analysis are tabulated in as:

Table 6: Deformation of differential gears

Sl.No	Gear name	Minimum (mm)	Maximum (mm)
1	Final drive gear	0	1.8321e-5
2	Crown gears	0	1.2678e-5
3	Side gears	0	1.2499e-5
4	Ring gear	0	3.8695e-6

The maximum displacement occurred during above deformations in differential gears are in permissible limits which can be determined by the properties of material. This differential is easy to assemble and disassemble.

For an auto racing application, where the driveline is routinely disassembled, inspected and repaired before and after race.

7.0 CONCLUSIONS

From the theoretical calculations and FEA analysis the following conclusions are drawn:

Maximum tangential force of 2126.127N, axial and radial force of 547.1929N obtained for Crown gears and Side gears. This is because of small size of the gear, but receives more torque. Maximum Effective load of 5825.962N acts on the Final and Ring gears that has Beam and Wear strength of 20,178.32N and 35,954.53N respectively. So, Factor of safety is required to keep the gears safe. The Maximum von-Misses obtained in Static structural analysis of Final, Crown, Side and Ring gears are far less than the Allowable stress of 550Mpa.Factor of safety obtained in theoretical calculations for Crown gears (6.9) and Side gears (6.9) are close to the Factor of Safety obtained from FEA for Crown gears (5.3794) and Side gears (6.7753). It is same in case of Final drive and Ring gears. The above results are valid only for a torque of 122N-m and speed of 4000rpm.In the present work, we considered single torque, speed and gear material 20MnCr5. However, the work can be extended to other materials at different torque and speeds.

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