IMAGE CLASSIFICATION OF TEMPERATURE DISTRIBUTION USING FOURIER SERIES STRATEGY

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ABSTRACT

This paper presents the analysis of temperature distribution by using Fourier series strategy. Using this strategy, two analyses have been made where an even and odd number is applied in the series. In even number, the trivial solution occur where there are no solution been made, but in odd number the solution is succeed. By using the equation made by odd number, the equation has been analysis using MatLab-Programming. In this analysis, the contour mapping of temperature distribution is shown clearly. Based on the mapping, it can be concluded that the temperature distribution happens because of the adiabatic phenomenon of the material properties itself.

KEYWORDS*: Fourier series, Isotherms, Adiabatic, Temperature distribution.*

1.0 INTRODUCTION

It is not easy to give an exact definition of temperature because majority of human around the world classify temperature to only familiar with hotness or coldness. In physiological ambiance, the level of temperature can be measured as a cold, freezing cold, warm, hot and red hot. For the basic example, metal will feel much colder than the wood when both materials are at the same temperature and same place. According to this example, several materials properties can be changed regarding the changes of the temperature around the material itself. The material properties can be changed with temperature in a repeatable and predictable way according the basis of accurate temperature measurement.

In a common example, a cup of hot tea present on the table ultimately cools off and cold drink sooner or later warms up. In this phenomenon, a body of hot tea and cold drink are brought into contact with another body with different temperature and heat transfer phenomenon is happen where the hot temperature transfer it temperature to the low temperature areas. This phenomenon will continue until both area reach with same temperature. When both areas are in a same temperature, these two areas are said in a thermal equilibrium condition. Zeroth Law of Thermodynamics explains that if two bodies in thermal equilibrium with a third body, they are also in thermal equilibrium for each others. This law had been formulated by R.H.Fowler in 1931.

In this study, the authors have studied the mediums' properties according to the heat transfer effect in several materials. Therefore, the knowledge on medium's value of temperature at all point is necessary. Heat transfer analysis basically plays a central role in the design of chemical processes and in the development of process system. In order to make accurate analysis to the heat transfer problem, parameters such as the roll speed, thermal conductivity, rate of cold air, thickness and temperature surface is needed.

Heat transfer problem can occur by three mechanisms and there are conduction, convection and radiation. Conduction mechanism is a collision of molecules causes the thermal energy to be transferred from one molecule to other one molecule. In this heat transfer process, the very energetic molecules will lose their energy while the lower energy molecules will get the more energy. In convection mechanism, it only occurs when the energy in macroscopic flow in fluid was associated with a parcel. Then, the fluid was converted to another region of space. In this case, it also can be called as an unsteady state behaviour. The radiation mechanism happens when the molecular vibrate and give an electromagnet radiation which certain amount to the other molecular. The radiation behaviour transmits the energy throughout the space and vacuum containing.

These three mechanisms have a potential to generate the instantaneous values of temperature at all points of the medium of interest and also called as a temperature distribution or field. The unsteady temperature distribution appears when medium's temperature not only varies from point to point, but also depends on time. In time domain, the temperature at a various points in a medium can be changed and then the internal energy of the molecular also changed. A steady state temperature distribution occurs when a temperature at a given point never varies with time and this type is called as a space coordinates only.

The temperature distribution by governing equations also called a three dimensional. This equation of temperature is describes as a function of three space coordinates, $r = \hat{\imath}x + \hat{\jmath}y + \hat{k}z$. Therefore, if the points of a medium with equal temperatures are connected, the resulting surfaces are called isothermal surfaces. This intersection of isothermal surfaces with a plane yields a family of isotherms on the place surface. Important to note that the two isothermal surfaces never cut each other since there are no part of the medium can have two different temperatures at the same time, respectively (Kreysig, 2006), (Howard *et al*., 2003). Fourier series of all dimension is a general type of summation process under which the convergence or non-convergence of the corresponding partial sums at a given point depend only on the behaviour of the function at given point, and that continuity of the function at the point is sufficient for convergence (Bochner, 1935). Fourier series has been used for solving many heat transfer problems. Maria and Power (2000) was developed an efficient BEM scheme for the numerical solution of two-dimensional heat problems. The double Fourier series was rewritten using Green function that obtained by the images method. The double Fourier series is use in the domain integral of the integral representation formula to transform such integral into equivalent surface integrals. Maksimovich and Tsybul''skii (2004) determined nonstationary nonaxisymmetric temperature fields in bodies of revolution appearing on heating by internal heat sources through and due to convective heat exchange with an external medium. The solution of the problem is represented in the form of a Fourier series in an angular coordinate with coefficients being determined by a method of boundary elements.

2.0 FOURIER SERIES STRATEGY

Fourier series is a one method to analyze the periodic phenomenon and they occur quite frequently in engineering and elsewhere such as in rotating of machines, alternating electric currents or the motions of planets. Fourier series is also called as a Trigonometric series and it represent in periodic functions. The function on trigonometric can be defined from $-\infty < x < \infty$, and has a period of 2π if $f(x+2\pi) = f(x)$ for all values of x. According to the definition of the periodic, most practical situations such as a function can be expressed as a complex Fourier series like (Kreysig, 2006), (Black, 2008):

$$f(x) = \sum_{j=-\infty}^{\infty} c_j e^{ijx} \quad where\ i = \sqrt{-1} \tag{1}$$

The number c_j is called as complex coefficients periodic functions. It can be compute by the integration as:

$$c_j = \frac{1}{2\pi}\int_0^{2\pi} f(x)e^{-ijx}\,dx \tag{2}$$

This Fourier series also can be written as sine and cosine function. The new equation of this Fourier series represents as a:

$$f(x) = c_0 + \sum_{j=1}^{\infty}(c_i - c_{-j})\cos(jx) + i(c_j - c_{-j})\sin(jx) \tag{3}$$

By the denoting the equation:

$$a_j = c_j + c_{-j} \quad and \quad b_j = i(c_j - c_{-j}) \tag{4}$$

Yields:

$$f(x) = \frac{1}{2}a_0 + \sum_{j=1}^{\infty} a_j\cos(jx) + b_j\sin(jx) \tag{5}$$

This series is called a Fourier sine-cosine expansion. For this case, $c_{-j} = \bar{c}_j$ for all values of j, which it is, implies that c_0 must be real and then:

$$a_j = 2 \quad real(c_j)|, \quad b_j = -2\, imag(c_j) \quad for\ j > 0 \tag{6}$$

Suppose that, Fourier series expansion for a more general function of and having a period of p instead of 2π . Then, the new function can be introduced by:

$$g(x) = f\left(\frac{px}{2\pi}\right) \tag{7}$$

This $g(x)$ has a period of 2π. This function of $g(x)$ can be rewrite to a new equation and representing as:

$$g(x) = \sum_{j=-\infty}^{\infty} c_j e^{ijx} \tag{8}$$

Where as:

$$f(x) = g\left(\frac{2\pi x}{p}\right) \tag{9}$$

Therefore the equation can be express into:

$$f(x) = \sum_{j=-\infty}^{\infty} c_j e^{2\pi ijx/p} \tag{10}$$

Sometimes, there occur functions that can be expanding as a series of a sine term only or as a series of cosine terms only.

In a case, if the function is originally defined for

$$0 < x < \frac{p}{2} \tag{11}$$

Therefore, it can be making as a

$$f(x) = -f(p-x) \qquad for \qquad \frac{p}{2} < x < p \tag{12}$$

This is a series where it involves only a sine terms. Similarly that, if

$$f(x) = +f(p-x) \qquad for \qquad \frac{p}{2} < x < p \tag{13}$$

This term consists only cosine term and finally, the equation can be represented as:

$$f(x) = c_0 + \sum_{j=1}^{\infty}(c_j + c_{-j})\cos\left(\frac{2\pi jx}{p}\right) \qquad if \qquad f(x) = f(p-x) \tag{14}$$

Whereas, this equation can be modifying as:

$$f(x) = \sum_{j=1}^{\infty} i(c_j + c_{-j})\sin\left(\frac{2\pi jx}{p}\right) \qquad if \qquad f(x) = -f(p-x) \tag{15}$$

Fourier series is a function of approximation using a finite number of terms and then, the resulting function may oscillate in regions where the actual function is discontinues or in other hand it is called as changes rapidly.

This undesirable behaviour can be reduced by using a smoothing procedure described by Lanezos where it use this Fourier series and close it related to a function of $\hat{f}(x)$ and it is defined by a local averaging process according to Equation 16 below.

$$\hat{f}(x) = \frac{1}{\Delta}\int_{x-\frac{\Delta}{2}}^{x+\frac{\Delta}{2}} F(\zeta)d\zeta \tag{16}$$

The averaging interval of Δ should be a small fraction of the period of p. Then, rewrite the equation of $\Delta = \alpha p$ with $\alpha < 1$. The functions of $f(x)$ and $\hat{f}(x)$ are identical as $\alpha \to 0$.

Hence, $\alpha > 0$ also match exactly at any point of x, where $f(x)$ varies linearly between

$$x - \frac{\Delta}{2} \qquad and \qquad x + \frac{\Delta}{2} \tag{17}$$

Importantly, this $\hat{f}(x)$ agreed closely with $f(x)$ for a small value of but at the same time in a Fourier series which converges more rapidly than the series for $f(x)$. Last but not least, it can be defined as:

$$\hat{f}(x) = \sum_{j=-\infty}^{\infty} c_j \frac{1}{p\alpha}\int_{x-\frac{\alpha p}{2}}^{x+\frac{\alpha p}{2}} e^{2\pi ijx/p}dx = \int_{j=-\infty}^{\infty} \hat{c}_j e^{2\pi ijx/p} \tag{18}$$

Where $\hat{c}_0 = c_0$ and $\hat{c}_j = c_j \sin(\pi j \alpha)/(\pi j \alpha)$ for $j \neq 0$. Then, evidently this Fourier series of the coefficients of $\hat{f}(x)$ are easily obtained from the function of $f(x)$. When this series for $f(x)$ converges slowly, using the same number of the terms in the series for $\hat{f}(x)$ often gives an approximation preferable to that provided by the series of $f(x)$. Finally this process is called as smoothing.

3.0 FOURIER SERIES ANALYSIS OF THIN SQUARE METAL PLATE

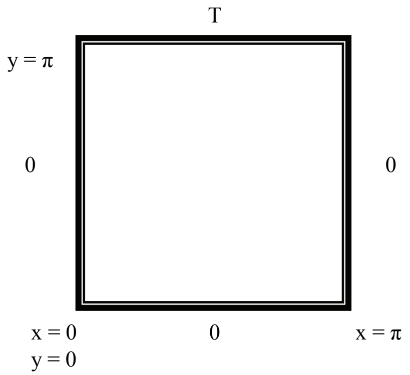

FIGURE 1
Plate

A thin square metal plate has three sides that are held at temperature 0. The other side (the top) is fixed at a temperature $T\,^{\circ}C$ above 0.
Using the boundary conditions:

$$u(0,y) = 0$$
$$u(\pi,y) = 0$$
$$u(x,\pi) = 0$$
$$u'(x,0) = 0$$

According to the Laplace Equation, it can be derived as a:

$$\nabla^2 u = \frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 0 \tag{19}$$

Let:

$$u(x,y) = F(x) \cdot G(y) \tag{20}$$

Since,

$$U_{xx} = -U_{yy}$$

Where,

$$\nabla^2 u = U_{xx} + U_{yy} = 0$$

From Equation 20, it can be rewritten as:

$$U_{xx} = G \cdot \frac{d^2 F}{dx^2} \tag{21}$$

$$U_{yy} = F \cdot \frac{d^2 G}{dy^2} \tag{22}$$

Re-arrange the equation and substitute Equation 21 and 22 into $\nabla^2 u = U_{xx} + U_{yy} = 0$. The new equation can be written as:

$$G \cdot \frac{d^2 F}{dx^2} + F \cdot \frac{d^2 G}{dy^2} = 0$$

$$G \cdot \frac{d^2 F}{dx^2} = F \cdot \frac{d^2 G}{dy^2} \tag{23}$$

From Equation 23, it needs to be divided with FG. Therefore, the equation can be expressed as:

$$\frac{1}{F} \cdot \frac{d^2 F}{dx^2} = -\frac{1}{G} \cdot \frac{d^2 G}{dy^2} \tag{24}$$

Based on Equation 24, equating the equation with where it is a constant value. Re-written of Equation 24 is:

$$\frac{1}{F} \cdot \frac{d^2 F}{dx^2} + \frac{1}{G} \cdot \frac{d^2 G}{dy^2} = -k$$

Hence,

$$\frac{1}{F}\cdot\frac{d^2F}{dx^2} = -k$$

And

$$-\frac{1}{G}\cdot\frac{d^2G}{dy^2} = -k$$

So,

$$\frac{d^2F}{dx^2} = -kF \quad or \quad \frac{d^2F}{dx^2} + kF = 0 \tag{25}$$

$$-\frac{d^2G}{dy^2} = -kG \quad or \quad \frac{d^2G}{dx^2} - kG = 0 \tag{26}$$

The left and the right side of the boundary conditions can be implied as:

$$F(0) = 0 \quad ; \quad F(\pi) = 0$$

It can give as a:

$$k = \left(\frac{n\pi}{\pi}\right)^2 = n^2 \qquad (This\ is\ only\ for\ non\ zero\ solutions)$$

Hence that,

$$F = F_n(x) = \sin nx \qquad where\ n = 1,2,3 \tag{27}$$

The Ordinary Differential Equations (ODEs) for G with $k = n^2$ can represent as a:

$$\frac{d^2G}{dy^2} - n^2G = 0 \quad or \quad \frac{d^2G}{dy^2} = n^2G$$

Then,

$$G_n(y) = A_n e^{ny} + B_n e^{-ny} \tag{28}$$

From Equation 28, differentiate respect to y by putting boundary condition

$$\frac{du}{dy} = 0$$

Obtained A = B. Therefore from Equation 28:

$$G_n(y) = A_n(e^{ny} + e^{-ny}) \tag{29}$$

From Equation 29, using Euler's formula in trigonometry, the equation can be expressed as:

$$A_n(e^{ny} + e^{-ny}) = 2A \cosh ny \tag{30}$$

Multiply both F and G, represent the equation as a:

$$U_n(x, y) = F_n(x) \cdot G_n(y) = 2A_n^* \cosh ny \cdot \sin nx \tag{31}$$

The boundary condition of $u(x, \pi) = T$ then is applied at the top side of the thin plate which subsequently the Equation 31 can be rewritten again as:

$$U_n(x, y) = F_n(x) \cdot G_n(y) = 2A_n^* \cosh n\pi \cdot \sin nx = T \tag{32}$$

$$U(x.\pi) = T = \sum_{n=1}^{\infty} 2A_n^* \cosh n\pi \sin nx \tag{33}$$

The final equation can represent as a:

$$U(x, y) = \sum_{n=1}^{\infty} U_n(x, y) = \sum_{n=1}^{\infty} 2A_n^* \sin nx \cosh ny \tag{34}$$

$$A_n^* = \frac{1}{\pi \cosh(n\pi)} \int_0^{\pi} T \sin nx \, dx \tag{35}$$

Then,

$$\int_0^{\pi} T \sin nx \, dx = \left[\frac{-T\cos nx}{n}\right]_0^{\pi} \tag{36}$$

If n is even number, n = 2, 4, 6, and 8… Equation 36 will become as:

$$\left[\frac{-T\cos nx}{n}\right]_0^{\pi} = \left[\frac{-T\cos(2\pi)}{2}\right] - \left[\frac{-T\cos(0)}{2}\right] = 0 \quad (trivial\ solution)$$

If n is odd number, n = 1, 3, 5, 7 …

$$\left[\frac{-T\cos nx}{n}\right]_0^{\pi} = \left[\frac{-T\cos(\pi)}{2}\right] - \left[\frac{-T\cos(0)}{2}\right] = 2T \qquad (non-trivial\ solution)$$

Therefore,

$$A_n^* = \frac{1}{\pi \cosh(n\pi)} \int_0^{\pi} T \sin nx \, dx$$

$$= \frac{T}{n\pi \cosh(n\pi)} [1 - \cos n\pi] \tag{37}$$

From Equation 37 and above, the general equation of the Fourier series of this problem can be representing as a:

$$U(x,y) = \sum_{n=1}^{\infty} U_n(x,y) = \sum_{n=1}^{\infty} 2\left(\frac{T}{n\pi \cosh(n\pi)} [1 - \cos n\pi]\right) \sin nx \cosh ny \tag{38}$$

When n = 1

$$U(x,y) = \sum_{n=1}^{\infty} U_n(x,y) = \sum_{n=1}^{\infty} 2\left(\frac{T}{\pi \cosh(\pi)} [1 - \cos \pi]\right) \sin x \cosh y \tag{39}$$

4.0 RESULT AND DISCUSSION

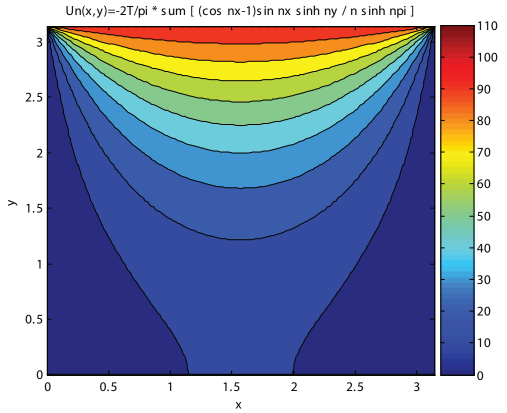

FIGURE 2
Temperature contour of thin square metal plate

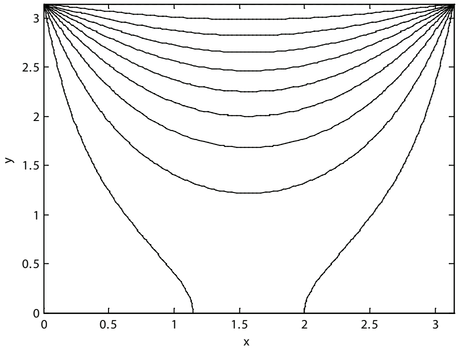

FIGURE 3
Isotherms line of thin square metal plate

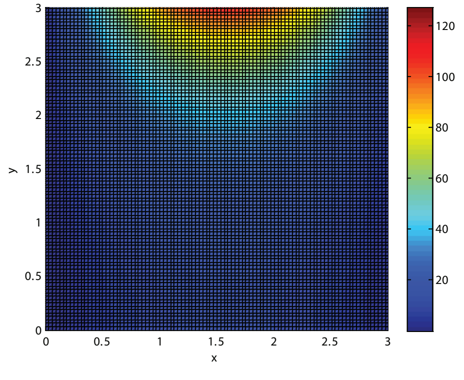

FIGURE 4
Two dimensional of thin square metal plate

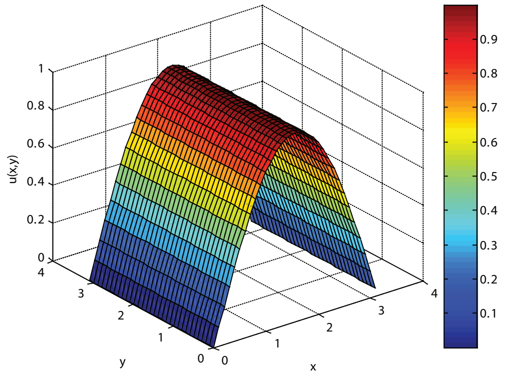

FIGURE 5
Three dimensional of thin square metal plate

Figure 2 shows the temperature contour of thin square metal plate. It can be seen that heat is transfer from high temperature to low temperature. The direction of heat transfer is due to temperature setting at the boundary condition. Left and right edges are set as cold (T=0) and the bottom edge set as an adiabatic boundary condition $\frac{dT}{dt} = 0)$ and the top edge is set at temperature T. Isotherm lines was plotted in Figure 3, it shows isotherm move out from the bottom edge. This is due the adiabatic boundary condition. At the left and right edges of thin plate, the isotherm lines keep parallel with these edges. This occurs due to left and right edges of thin plate was set as a cold (T=0). Figure 4 and Figure 5 show the two and three dimensional of temperature distribution of thin square metal plate. Figure 5 clearly shows the steady state heat transfer conditions. At steady state condition, heat transfer from the top edge of thin square metal plate is equal to heat transfer out from bottom edge of thin square metal plate.

5.0 CONCLUSION

Fourier series analysis is also known as a trigonometric series analysis. This series has a potential to solve the analytical solution on temperature distribution of plate.

Temperature distribution can be distributed independently according to the adiabatic phenomenon. This phenomenon is based on the time domain.

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