COMBINATION OF CFD AND MONTE CARLO SIMULATION TO INVESTIGATE THE NONLINEAR BEHAVIOR OF A NON-AGING VISCOELASTIC PLATE IN SUBSONIC FLOW

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ABSTRACT

Usually in studying the conventional fluid-solid interaction problems, both the flow field and the solid structure must be jointly meshed and solved. This takes a huge amount of iteration and time for calculation even for simple specific examples. One of the most industrial elements used in fuselage of aero-space systems is the plate whose instability and behavior, especially in the case of large deformation, is so vital due to its effect on the overall performance of the system. In this paper, utilizing a new method that combines the CFD and Monte Carlo simulation, the nonlinear behavior of a two dimensional simply supported non aging viscoelastic plate located in a subsonic flow is investigated. First, relative to the plate boundary conditions, the whole behavior of the plate is estimated. Then, using CFD simulation, the flow field is solved for some various plate deformations. This prepared a bank of data for the domain of plate response. Due to the dynamic behavior of a turbulent flow which presents highly nonlinear terms and disturbances; the aerodynamic forces are modeled by some random forcing functions using statistical procedure. Finally, using Monte Carlo simulation used for randomly excited ODEs, the forces evaluated from CFD for each deformation are applied to the nonlinear equation of motion of the plate and the behavior and possible instabilities are investigated.

KEYWORDS: *Fluid-solid interaction; Random vibration; Viscoelastic plate; Non-aging material; Monte Carlo simulation; Nonlinear behavior investigation*

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1.0 INTRODUCTION

The dynamic behavior and instability phenomenon such as divergence and flutter are the most important design considerations for a fluid-solid interaction system. Plates as one of the best applicable industrial elements have been vastly used in these structures whose responses must be carefully checked. In the case of nonlinear bi-directional or two ways FSI, the problem divided to two major parts: simulation and solving the fluid flow field and then the nonlinear behavior and instability recognition of the plate as the structure. Many simulations can be found in literature used famous FSI methods such as loosely (one way) and strongly (two ways) coupled FSI methods and solved these two parts jointly. See for example (Du, 2010), (Deiterding et al., 2008), (Deiterding, 2010), (Paidoussis, 1998), (Shishaeva et al., 2012), (Urikovi et al., 2005), (Wong, 2011).

Since in these FSI simulations, both the flow field and the structure must be meshed and solved, we face with a huge amount of iteration and calculation time ever for specific case
studies whose flow field and structure properties such as material and geometry were fully determined. In this research we investigated the nonlinear behavior of a two dimensional simply supported plate located in a subsonic flow by dividing the whole FSI problem to two simpler and applicable sub-problems.

First, relative to the external supports, the normal modes of the plate are obtained. The whole behavior of the plate can be estimated from these normal modes as some admissible functions (Rao, 2004). Second, using CFD simulation, the flow field is solved for some various structure deformations obtained previously from the normal mode functions of the plate. This prepared a bank of data for the domain of plate response. Third, regard to the dynamic behavior of a turbulent flow which presents highly nonlinear terms and disturbances; the aerodynamic forces are modeled by some random forcing functions using statistical procedure.

Finally, relative to the works have been done in studying the behavior of plates under random forcing functions such as Monte Carlo simulation and using an interpolation code, the forces evaluated from CFD for each deformation are applied to the nonlinear equation of motion of the plate and the behavior and possible instabilities are investigated.

Historically, the basic studies on Conservative elastic systems under random forces can be found in literature at the 1950 and 1960 decades

(Potapov, 1999). This reference also said that the behavior of the viscoelastic material, especially against accidental loads due to their molecular structure becomes random and nonlinear with respect to time. For recent studies, see (Sun, 2001) and (Eshmatov, 2007).

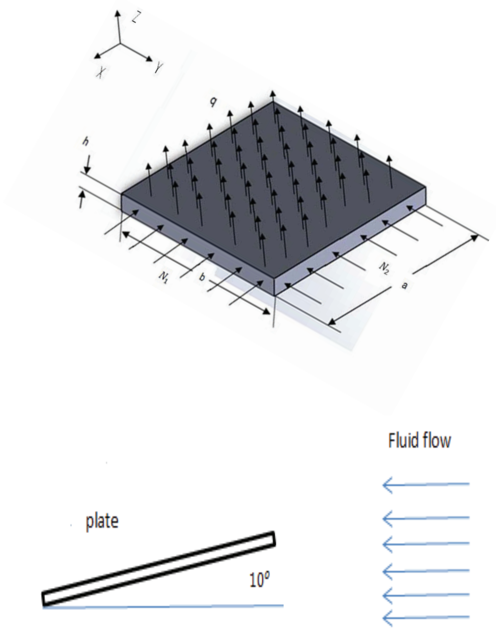
Simulation of plates under random forces has been done in literature using different methods. They are based on to use FPK equation to obtain exact solutions for both linear and nonlinear systems (Bolotin, 1984; Cai et al., 1996; Potapov, 1999), statistical linearization to obtain some local behavior of nonlinear systems (Roberts et al., 1990), the dynamics of statistical moments to obtain semi-exact solutions for both linear and nonlinear systems (Bolotin, 1984 and Roberts, 1990) and Monte Carlo simulation as one of the numerical solutions for such systems (Potapov, 1999; Bolotin, 1984; Asnafi, 2001). The behavior investigation of plates and panels due to aerodynamic forces can also be found in literature using different methods. One of the earlier works, for example, is one done by Dowell (1970) that analyzed the problem of linear elastic plate under the aerodynamic forces. One of the most popular methods in earlier studies was to use piston theory to model the aerodynamic forces acting on a plate. Recently and due to the development of numerical simulation softwares, solving the fluid solid interacting problems has increased significantly. See for example (Du, 2010), (Deiterding et al., 2008) also (Franco et al., 2008), (Bartoli, 2006), (Eloy et al., 2007), (Giordano et al., 2005), (Kambouchev et al., 2007), (Song et al., 2011), (Wulf et al., 2013). These FSI simulations, especially for nonlinear systems take a huge amount of time for calculation.

In this article, we combine two previous methods to investigate the behavior of nonlinear plates. In other words, the fluctuating forces obtained by CFD are first modeled by random variables and then fed to a code written to solve nonlinear plates under the equations of fluid flow. To obtain the dynamic behavior of the plate, first, we must solve the equations of fluid flow in several deflections of plate to obtain an aerodynamic force bank and its corresponding random model. The latter will be sent to the Monte Carlo simulation written to investigate the nonlinear behavior of plates under random forcing functions.

2.0 MATHEMATICAL EQUATIONS FOR VISCOELASTIC PLATE AND FLUID FLOW

2.1 Governing equation of the viscoelastic plate in large deformation

The governing equation of motion for a viscoelastic plate in large deformation is taken from (Asnafi, 2001) which is known as the Von Karman equations (see also Figure 1).



(a) (b)

Figure 1. (a) The schematic geometry of plate and acting forces and (b) The front view of the plate and fluid flow

$$\nabla^4 \Phi = E(I - R)(w_{xy}^2 - w_{xx} w_{yy}) \tag{1}$$

$$D(I - R)\nabla^4 w = q + h[\Phi_{yy} w_{xx} - 2\Phi_{xy} w_{xy} + \Phi_{xx} w_{yy}] \tag{2}$$

where

Φ denotes the airy stress function that must be calculated from the boundary conditions and in-plane forces
a is the length
b is the width and h is the thickness of plate
w is the deflection

k is the assumed external damping coefficient
q is the lateral distributed load,
E is the young's modulus of elasticity
D is the flexural stiffness
p is the density per unit area of plate
I is the identity operator
R is the relaxation kernel

In this paper, we consider two dimensional simply supported plate and a general non-aging viscoelastic material whose relaxation kernel can be assumed as (Potapov, 1999).

$$R_0(t-\tau) = \chi L e^{-\chi(t-\tau)} \tag{3}$$

where χ and L are the viscoelastic material properties. Here, and to solve the examples in this article, the numerical values of these article are taken from (Potapov, 1999) which are both equal to 0.3.

One of the best methods used to solve the partial differential equation of the continuous vibratory system is the Galerkin's method (Rao, 2004). Here, the deflection can be written by series of eigen-functions as:

$$w(x,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \bar{W}_{mn}(t)\sin(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b}) \tag{4}$$

For the first vibration mode, the deflection w can be expressed as:

$$w = \bar{W}\sin(\frac{\pi x}{a})\sin(\frac{\pi y}{b}) \tag{5}$$

Now, after computing the airy stress function, one can multiply the obtained relation to $\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$ and then integrate on the domain and reach to:

$$x'' + 2\varepsilon x' + (I-R)x - \alpha\left(N_x + \frac{a^2}{b^2}N_y\right)x + \beta x(I-R)x^2 = P_r \tag{6}$$

where

$$\beta = \frac{3}{4}(1-\upsilon^2)\frac{a^4+b^4}{(a^2+b^2)^2}, \quad P_r = \frac{16a^4b^4}{\pi^6 hD(a^2+b^2)^2}q, \quad 2\varepsilon = \frac{k}{\rho h\omega}, \quad \alpha = \frac{\pi^2}{Da^2}(\frac{\pi^2}{a^2}+\frac{\pi^2}{b^2})^{-2};$$
$$\omega^2 = \frac{D}{\rho h}(\frac{\pi^2}{a^2}+\frac{\pi^2}{b^2});$$
$$\tau = \omega t, \quad x = \frac{\bar{w}}{h}, \quad (.)' = \frac{\partial}{\partial\tau}, \tag{7}$$

and N_x, N_y are non in-plane forces along x and y direction respectively.

All the parameters introduced in Equation (6) relate to plate properties except the scaled lateral and in-plane forces that must be evaluated from CFD. Here, of course, we face with a stochastic ODE that must be simulated and solved via one of the appropriate simulators such as Monte Carlo simulation.

This simulation is computational algorithm that relies on repeated random sampling to obtain numerical results and is useful for obtaining numerical solutions to stochastic problems which are too complicated to solve analytically (Fishman, 1995). They are often used in simulating systems with many coupled degrees of freedom, such as fluids, disordered materials, strongly coupled solids, and cellular structures and are most suited to be applied when it is impossible to obtain a deterministic algorithm. Here and in this article, we use this simulation by a written code in MATLAB to generate the random data and solve the ODEs properly.

2.2 GOVERNING EQUATIONS OF FLUID FLOW

Governing equations for ideal compressible flow that are considered by the software are continuity equation, momentum, energy and the equation of state as presented below. Knowing that the flow is turbulent, we consider the Spalart-Allmaras model designed especially for aerospace applications involving wall-bounded flows and has been shown to give good results for boundary layers subjected to adverse pressure gradients (Anderson, 1995).

$$\frac{\partial \rho}{\partial t} + div(\rho \vec{V}) = 0 \tag{8}$$

$$\rho \frac{D\vec{V}}{Dt} = \rho \vec{g} + \vec{\nabla}\left(-p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \gamma div(\vec{V})\delta_{ij}\right) \tag{9}$$

$$\rho \frac{Dh}{Dt} = \frac{Dp}{Dt} + div(k\vec{\nabla} T) + \left(\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \gamma div(\vec{V})\delta_{ij}\right)\frac{\partial u_i}{\partial x_j} \tag{10}$$

$$p = \rho RT \tag{11}$$

These four equations are solved to obtain the flow field. As indicated previously, in the method presented in this article, these equations must be solved for different deflections of plate. Using this approach and to reduce the amount of calculation, we choose steady solution which shows completely admissible errors with respect to non steady full FSI simulator ,see Table 1.

Table 1. Deflection rate of the plate and the flow velocity along z direction

Mach number	Max. deflection rate of the plate (m/s)	Flow Velocity magnitude(m/s)	Flow velocity along z direction (m/s)
0.3	0.0731	104.1263	18.0813
0.4	0.0728	138.8351	24.1085
0.6	0.0720	208.2526	36.1627
0.8	0.0709	277.6702	48.2169

3.0 THE NONLINEAR BEHAVIOR OF THE CONVENTIONAL VISCOELASTIC PLATE

Since Equation 6 is transformed to a non dimensional format, it can be used to study the nonlinear behavior of several plates. Here, to obtain more feasible results, a specific geometry for the plate is chosen and to prepare the bank of data, the flow field is obtained for four values of Mach numbers including ten different types of deflection of the plate. Note also that to ensure about the results regardless of the number of grids, four different grid numbers, i.e. 1 million, 1.38 million, 1.7 million and 2 millions are considered but finally, and due to negligible changes in the flow fields larger than the 1.38 million grids, it is selected. In table 2, the properties of the solved plate are tabulated.

Table 2. Properties of the plate 1

a (m)	b (m)	h (m)	Young's modulus of Elasticity (Pa)	Poisson ratio
1	1	0.1	70×10^{9}	0.34

After the bank of data is obtained, using an interpolation process, the needed statistical properties are extracted, evaluated and applied as random forcing functions to the Monte Carlo simulation. The latter simulator, now, computes the statistical properties of the response of the plate.

3.1 Results for the flow its Mach equals to 0.3

The examples are solved for the initial velocities corresponding to each Mach and atmospheric pressure for the total domain. Figure 2 shows the mesh created to solve CFD for the case when the middle point deflection of the plate is equal to 0.04.These deflections are arranged with respect to the normal modes of plate, i.e. those presented in Equation 4.

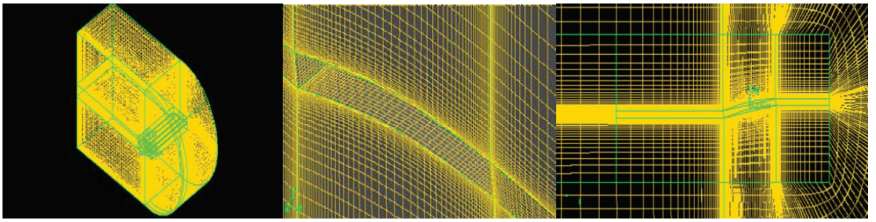


Figure 2. (a) Typical isometric view of the mesh used to solve the CFD, (b) Zoomed area of mesh on the plate and (c) Front view of the mesh

Solving the CFD code, the aerodynamic forces obtained along x (opposite flow) and z direction for different Mach numbers, themselves, including some deflections shown in Tables 3 to 6. See also Figure 1. Generally, if the angle of attack is denoted by θ (which is equal to 10^0 in this paper), the lateral and in-plane forces applied to plate becomes:

$$N_x = N_{xx}\cos(\theta) + N_{xz}sin(\theta) \tag{12}$$

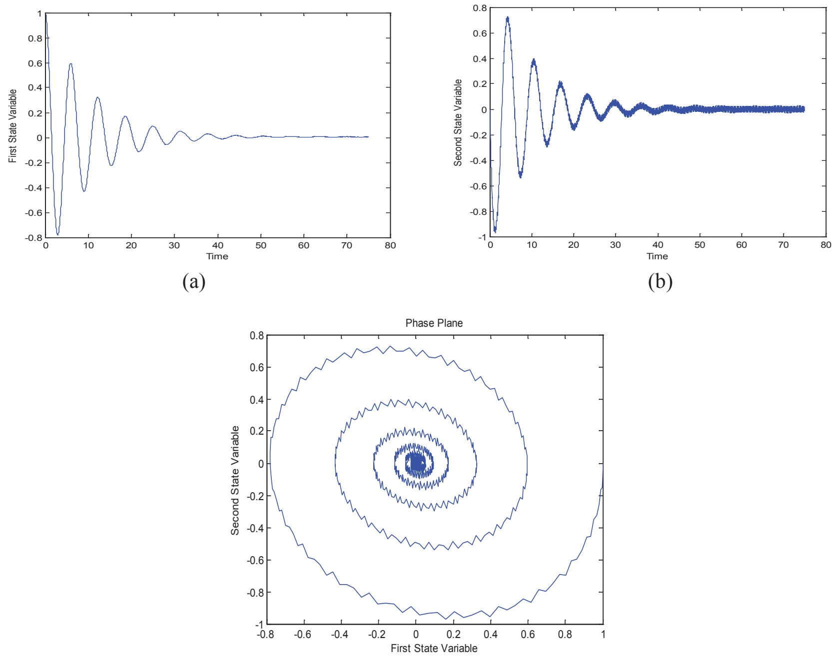
$$q = q_z\cos(\theta) + q_x sin(\theta) \tag{13}$$

$$N_y = 0\,, symmetric\ condition \tag{14}$$

In each case, processing these forces and applying them to simulator, the behavior of the plate is obtained. See the evolution of the dimensionless displacements, their rates and the phase plane diagrams in Figures 3 to 7.

Table 3. Applied aerodynamic forces of example 3.1 obtained from CFD

q_z (N)	$-N_{xz}$ (N)	q_x (N)	N_{xx} (N)	Deflection (m)
424.43446	59.004004	335.61873	192.06947	0.10
526.90734	58.787248	334.42897	184.20924	0.08
648.0294	59.339366	337.59522	179.89264	0.06
780.0557	60.897347	346.50478	180.71503	0.04
900.57862	60.408549	343.85178	181.84481	0.02
1144.82	62.404858	355.3536	194.73952	-0.02
1258.0741	63.461222	361.45684	206.31312	-0.04
1345.0368	63.866414	363.83924	217.62918	-0.06
1435.405	64.109654	365.5119	234.50085	-0.08
1494.1055	64.76591	369.29392	250.47391	-0.10



(a) (b)

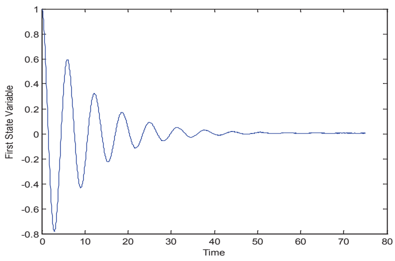
(c)

Figure 3. The evolution of (a) dimensionless displacement w.r.t scaled time, (b) dimensionless displacement rate w.r.t scaled time and (c) The phase plane of the behavior of the plate of example 3.1

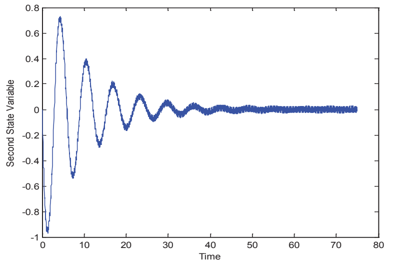
3.2 Results for the flow its Mach equals to 0.4

Table 4. Applied aerodynamic forces of example 3.2 obtained from CFD

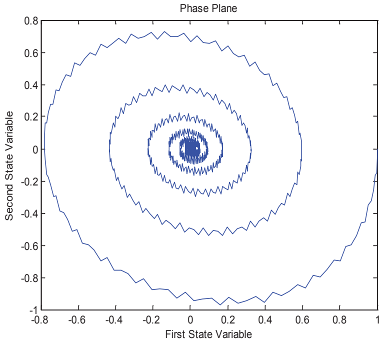
q_z (N)	$-N_{xz}$ (N)	q_x (N)	N_{xx} (N)	Deflection (m)
424.43446	59.004004	335.61873	192.06947	0.10
526.90734	58.787248	334.42897	184.20924	0.08
648.0294	59.339366	337.59522	179.89264	0.06
780.0557	60.897347	346.50478	180.71503	0.04
900.57862	60.408549	343.85178	181.84481	0.02
1144.82	62.404858	355.3536	194.73952	-0.02
1258.0741	63.461222	361.45684	206.31312	-0.04
1345.0368	63.866414	363.83924	217.62918	-0.06
1435.405	64.109654	365.5119	234.50085	-0.08
1494.1055	64.76591	369.29392	250.47391	-0.10



(a)



(b)



(c)

Figure 4. The evolution of (a) dimensionless displacement w.r.t scaled time (b) dimensionless displacement rate w.r.t scaled time and (c) The phase plane of the behavior of the plate of example 3.2.

3.3 Results for the flow its Mach equals to 0.6

Table 5. Applied aerodynamic forces of example 3.3 obtained from CFD

q_y (N)	$-N_{xy}$(N)	qx(N)	N_{xx} (N)	Deflection (m)
1446.1149	256.58704	1458.273	704.99802	0.10
1925.2559	254.87607	1448.5392	685.07758	0.08
2449.2704	257.13927	1461.4588	671.27993	0.06
3029.2859	263.23413	1496.2571	686.3058	0.04
3504.0315	262.84202	1494.4071	692.69649	0.02
4557.7717	269.02963	1529.8395	782.98761	-0.02
4978.0005	270.86306	1541.3522	835.64312	-0.04
5296.2103	274.01818	1559.0976	879.40922	-0.06
5606.6839	274.61812	1562.6258	933.10632	-0.08
5941.2815	277.39302	1579.0023	1010.3086	-0.10

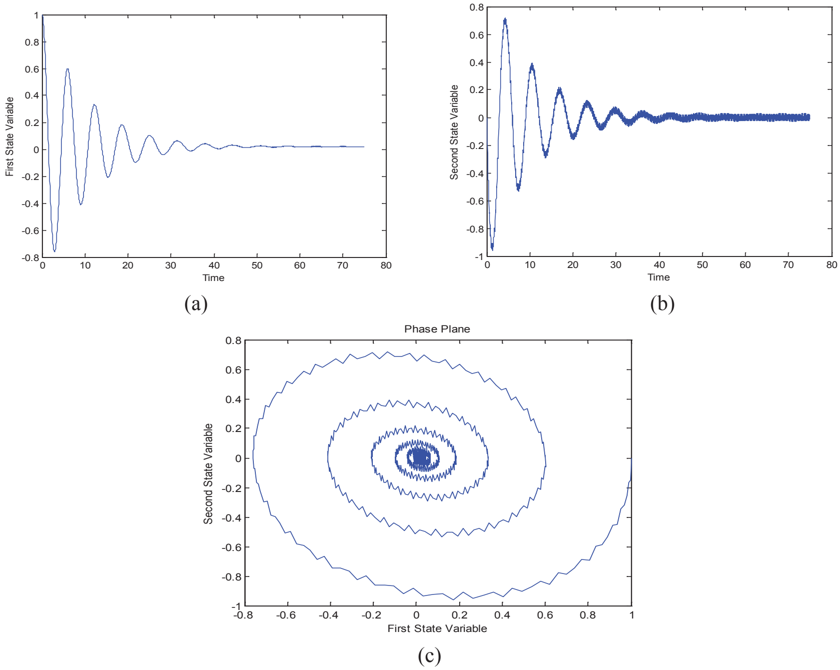


Figure 5. The evolution of (a) dimensionless displacement w.r.t scaled time, (b) dimensionless displacement rate w.r.t scaled time and (c) The phase plane of the behavior of the plate of example 3.3

3.4 Results for the flow its Mach equals to 0.8

Table 6. Applied aerodynamic forces of example 3.4 obtained from CFD

q_y (N)	$-N_{xy}$(N)	q_x(N)	N_{xx} (N)	Deflection (m)
2005.8174	498.84449	1232.212	2833.7934	0.10
3266.8832	492.41721	1174.6639	2797.0888	0.08
4193.2579	502.72698	1155.5163	2856.0791	0.06
5165.3032	514.39779	1158.0633	2922.1772	0.04
6372.4311	507.33931	1161.6569	2881.2215	0.02
8194.661	516.97404	1482.3084	2937.3178	-0.02
8926.2845	528.71467	1576.7716	3004.5875	-0.04
9486.2491	533.86457	1615.4845	3034.351	-0.06
9872.7652	528.88778	1668.7831	3006.9955	-0.08
10636.273	532.28926	1806.6512	3027.2057	-0.10

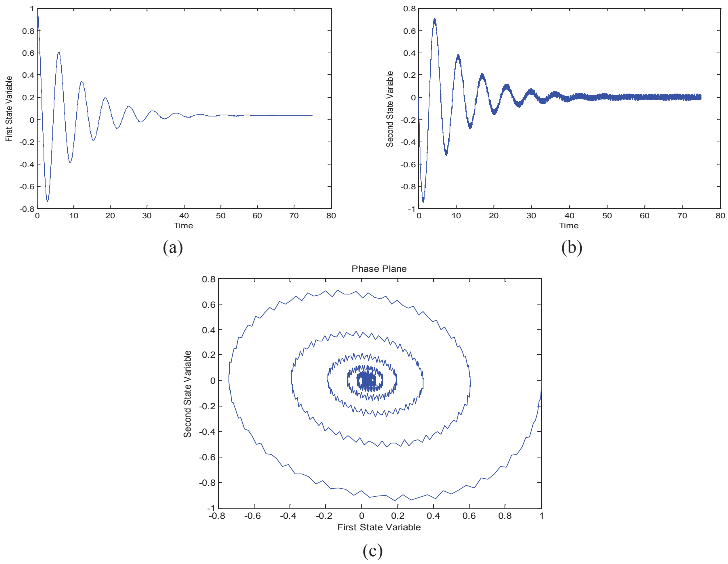


Figure 6. The evolution of (a) dimensionless displacement w.r.t scaled time, (b) dimensionless displacement rate w.r.t scaled time and (c) The phase plane of the behavior of the plate of example 3.4

As drawn in Figures (3-6), in all cases, the response (deflection) of the plate shows stable focus in the origin of the phase plane which means that the nonlinear vibration of plate decays in an oscillatory manner about origin. In Equation (6), if the dynamic terms vanish, we face with a third order polynomial, in general, has three roots that represent the equilibrium points of oscillator. Here, the origin of the phase plane ($x = 0$) is the dominant and of course demanded equilibrium point of the oscillator see Equation (6). This shows a good agreement with that was found in literature for random vibration of plates under laterally random forcing function (see for example Asnafi (2011 and 2012)).
In subsonic flow and relative to this thickness of plate, any other qualitatively different behaviors were not seen. In other words, the plate converges to its dominant equilibrium point located at the origin of the phase plane. In what follows, the procedure is repeated for a thinner plate to check about other nonlinear behaviors or instabilities.

4.0 THE NONLINEAR BEHAVIOR OF A THINNER VISCOELASTIC PLATE

The properties of the plate used to investigate the following examples are tabulated in Table 7.

Table 7. Properties of the thinner plate

a (m)	*b* (m)	*h* (m)	Young's modulus of Elasticity (Pa)		Poisson ratio
1	1	0.02	example 4.1	70×10^9	0.34
			example* 4.2	2.5×10^6	

*Taken from ANSYS CFX Tutorials (ANSYS Cooperation, 2010)

4.1 Example 4.1

Similar to that was done in section 3, the flow field is solved for different Mach numbers and then the behavior of the plate is obtained. Here to avoid prolixity, only the final behavior of the plate in each Mach number is reported.

Relative to the obtained results, it was recognized that in all case studies and Mach numbers (0.3 to 0.8) the behavior presents stable focus. Here to avoid prolixity, only the final behavior of the plate in each Mach number is reported by phase plane (see Figure 7).

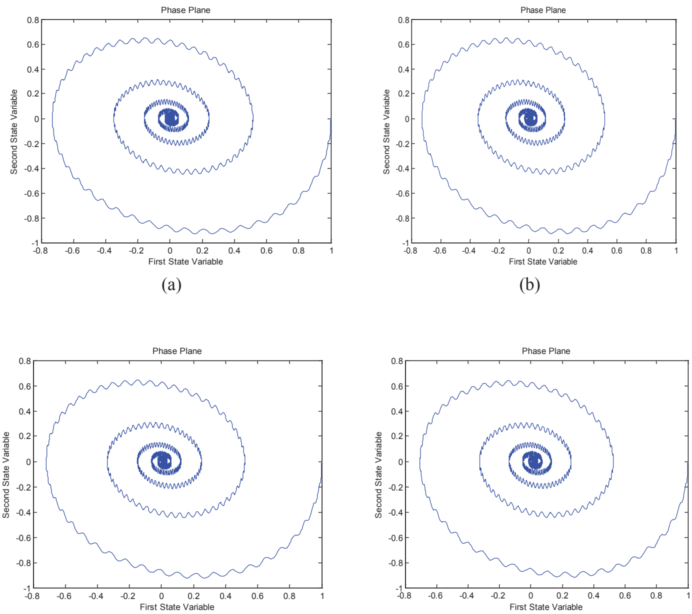


Figure 7. Phase plane of evolution dimensionless displacement versus dimensionless displacement rate for the flow its mach equals to (a) 0.3, (b) 0.4, (c) 0.6, and (d) 0.8

4.2 Example 4.2

Here the procedure is repeated but it was recognized that in all case studies and Mach numbers (0.3 to 0.8) the behavior becomes completely unstable. See Figure 8 which show flutter instabilities at different mach numbers.

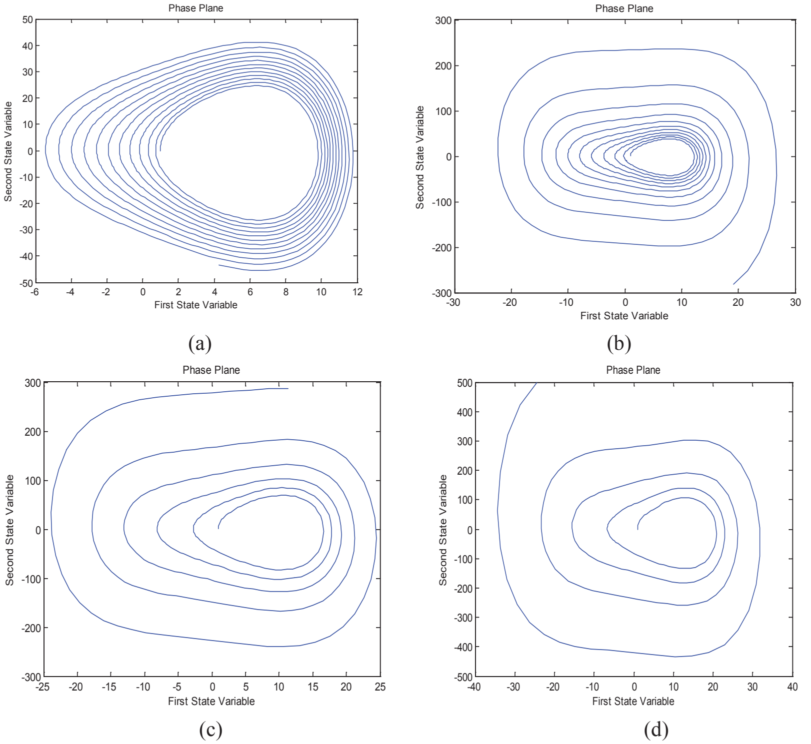


Figure 8. The phase plane diagram of the plate of example 4.2 at Mach number equals to (a) 0.3 (b) 0.4, (c) 0.6 and (d) 0.8

To validate the results obtained by this method and other conventional methods presented in literature for randomly excited plates, an analogy between the results obtained by the code written in (Asnafi, 2001) for randomly exited plate whose statistical properties are the same as those for the plate located in subsonic flow (M = 0.3) is held. See Figure 9 for more details.

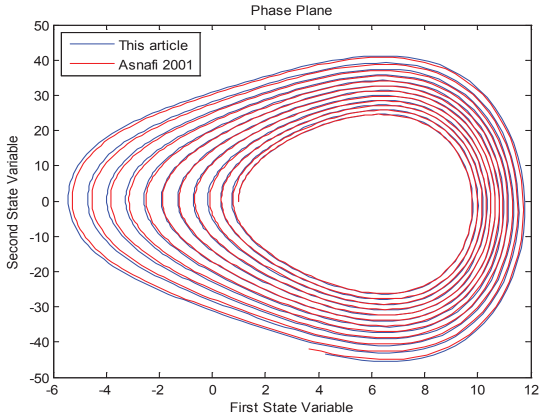


Figure 9. The analogy between the phase plane diagram of the plate solved in this article at Mach equals to 0.3 and one obtained by (Asnafi, 2001) for randomly excited plate with corresponding data

5.0 A DISCUSSION ON THE TIME OF SOLUTION

Here, a simple analogy between the time consumed by the method presented in this article and the conventional FSI method in Ansys CFX is made. Generally, the unsteady simultaneous solution of fluid structure interaction in ansys CFX for simply supported plate shows weak convergence and CFX linear solver fails in many cases. Of course the plate with fixed supports in incompressible flows can reach a good convergence, but in compressible flows it also suffers from weak convergence. In other words, FSI two-dimensional incompressible flow solution around a plate with fixed supports is more expensive than three-dimensional compressible flow solution around a plate with simple supports simulated by present method. Note also that the vibration of plate with fixed supports is decayed faster than the same plate with simple supports. In Table 8, these method are compared which shows a relatively good time consumption for the present method.

Table 8. Compared solution time of statistical method and the FSI method (ANSYS CFX)

Solution method	Flow velocity (m/s)	Mach number	Solution Algorithm	Flow geometry	Turbolent model	Flow type	Flow	Grid number	Solution of plate dynamic	Solution time (hours)
Present work	104.12	0.3	SIMPLE	3D	Spalart Allmaras	Compressible	Steady	1.38*10^6	Monte carlo simulation	Approximate 8
FSI in ANSYS CFX	15	-	COUPLED	2D	SST K-ω	In-compressible	Unsteady	Less than 30000	Finite Element	Approximate 10
FSI in ANSYS CFX	110	-	COUPLED	2D	SST K-ω	Compressible	Unsteady	Less than 30000	Finite Element	Approximate 30

6.0 CONCLUSIONS

In this paper, a method to investigate the behavior of non-aging viscoelastic plates in sub sonic flow as an example of the fluid-solid interaction problems is presented. The method especially is applied to a two dimensional simply supported viscoelastic plate located in the subsonic flow. After choosing the most efficient mesh grids, the aerodynamic forces are obtained by CFD and then modeled by stochastic variables and finally applied to Monte Carlo simulation which is arranged to solve randomly excited nonlinear ODEs. After the method is validated, the nonlinear behaviors of a conventional and an almost thin non-aging viscoelastic plate are obtained. The results obtained by this method can predict some famous nonlinear behavior and instabilities such as bifurcation in the plate response. Specifically it is shown that in subsonic flow and for a plate whose aspect ratio is less than or equal to 10, the behaviors for different Mach numbers exhibits stable foci which means that the oscillations decay about dominant equilibrium point oscillatory. Of course, for thinner plate, i.e. when the flexural stiffness decreases, the instability such as flutter due to bifurcation may occur also. Finally a discussion on the time of solution is presented shows the current method consumed specifically less time than other conventional FSI methods.

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