VIBRATION ANALYSIS OF SIZE-DEPENDENT NANOBEAMS BASED ON NONLOCAL TIMOSHENKO BEAM THEORY

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ABSTRACT

In the present paper, the semi-analytical differential transformation method (DTM) is employed for vibration analysis of size-dependent nanobeams based on nonlocal Timoshenko beam theory (TBT). The governing motion equations of nanobeam with different edge conditions are derived by the Hamilton's principle. DTM is applied to discretize the governing equations and boundary conditions, which are then solved to obtain the frequency parameters of nanobeam. In the numerical examples, the good agreements between the present results and existing literature verified the validity and accuracy of the present solution method. The detailed mathematical derivations are presented and numerical investigations are performed while the emphasis is placed on investigating the effect of small scale parameters, mode number, aspect ratios and edge conditions on the normalized natural frequencies of the nanobeams. It is explicitly shown that the vibration of a nanobeam is significantly influenced by these effects.

KEYWORDS: *Differential transformation method; Nonlocal Timoshenko beam theory; Vibration of nanobeams*

INTRODUCTION

In recent years, nanomechanical and nano-electro-mechanical systems at nanoscale receive special attention from researchers. Among all of them nanobeams attract more attention because of their potential usage and superior properties. Many recent experimental results have shown that as the size of the structures reduces to micro/nanoscale, the influences of atomic forces and small scale play a significant role in mechanical properties of these nanostructures (Chong et al., 2001). Thus, neglecting these effects in some cases may results in completely incorrect solutions and hence wrong designs. The classical continuum theories do not include any internal length scale. Consequently, these theories are expected to fail when the size of the structure becomes comparable with the internal length scale. Eringen nonlocal theory is one of the well-known continuum mechanics theories that includes small scale effects with good accuracy to model micro/nanoscale devises (Eringen & Edelen, 1972). The nonlocal elasticity theory assumes that the stress at a point is function of the strain at all neighbor points of the body, hence, this theory could take into account the effects of small scales.

In recent years, the studies of nanostructures using the nonlocal elasticity theory have been an area of active research. Based on this theory, Reddy (2007) derived the equation of motion of Euler–Bernoulli, Timoshenko, Reddy and Levinson beam theories and presented analytical and numerical solutions for deflections, buckling loads and natural frequencies of nanobeams. Moreover, bending, buckling and free vibration of nanobeams based on different beam theories investigated by Aydogdu (2009). Recently, Thai (2012) studied bending, buckling and vibration of nanobeams employing analytical methods. Most recently, in a similar work, free vibration of single walled carbon nanotubes with various edge conditions is examined by Ansari and Sahmani (2012).

The governing motion equations are often solved by analytical method which usually only utilize for simply supported edge conditions (Reddy, 2007; Aydogdu, 2009; Thai, 2012), or by finite element methods or generalized differential quadrature method (Ansari & Sahmani, 2012) and other solution methods which need high CPU time to solve. But in the present work we employ a novel semi-analytical method called differential transformation method which was first introduced by Zhou (1986) for solving linear and nonlinear initial value problems in electric circuits. The main advantage of this method is that it can be applied directly to partial differential equations without requiring linearization, discretization, or perturbation. It is a semi-analytical-numerical technique that formulizes Taylor series in a very different manner. By using this method, the governing differential equations can be reduced to recurrence relations and the boundary conditions may be transformed into a set of algebraic equations. It is different from the high-order Taylor series method which requires symbolic computation of the necessary derivatives of the data functions. Another important advantage is that this method reduces the size of computational work while the Taylor series method is computationally time-consuming especially for high order equations.

As seen, to the author's best knowledge there is no work on vibration analysis of nanobeams using TBT with differential transformation method for various edge conditions. Therefore, Timoshenko beam theory is employed based on Eringen's nonlocal elasticity Theory to consider the size-effect in free vibration analysis of nanobeams corresponding to four commonly used edge conditions including Simply supported-Simply supported (S-S), Clamped-Simply supported (C-S), Clamped-Clamped (C-C) and Clamped-Free (C-F). Then, DTM is utilized to detemine the natural frequencies of nanobeams. To illustrate the accuracy of present method, the obtained results are compared with those published works. Hence, The influences of the nonlocal parameter, aspect ratio and different edge condition on the free vibration characteristics of the nanobeams are discussed in details.

BASIC FORMULATIONS

2.1 Nonlocal Elasticity Theory

The constitutive equation of classical elasticity is an algebraic relationship between the stress and strain tensors while that of Eringen's nonlocal elasticity involves spatial integrals which represent weighted averages of the contributions of strain tensors of all points in the body to the stress tensor at the given point (Eringen, 2002). Though it is difficult mathematically to obtain the solution of nonlocal elasticity problems due to the spatial integrals in constitutive equations, these integropartial constitutive differential equations can be converted to equivalent differential constitutive equations under certain conditions. The theory of nonlocal elasticity, developed by Eringen & Edelen (1972) states that the nonlocal stress-tensor components σ_{ij} at any point x in a body can be expressed as:

$$\sigma_{ij}(x) = \int_{\Omega} \alpha(|x' - x|, \tau)\, t_{ij}(x')\, d\Omega(x') \tag{1}$$

where $t_{ij}(x')$ are the components of the classical local stress tensor at point x, which are related to the components of the linear strain tensor ε_{kl} by the conventional constitutive relations for a Hookean material, i.e:

$$t_{ij} = C_{ijkl}\varepsilon_{kl} \tag{2}$$

Equation (1) states that the nonlocal stress at point x is the weighted average of the local stress of all points in the neighborhood of x, the size of which is related to the nonlocal kernel $\alpha(|x'-x|,\tau)$. Here $|x'-x|$ is the Euclidean distance and τ is a constant given by:

$$\tau = \frac{e_0 a}{l} \tag{3}$$

which represents the ratio between a characteristic internal length, a (such as lattice parameter, C–C bond length and granular distance) and a characteristic external one, l (e.g. crack length, wavelength) trough an adjusting constant, e_0, dependent on each material. The magnitude of e_0 is determined experimentally or approximated by matching the dispersion curves of plane waves with those of atomic lattice dynamics. For a class of physically admissible kernel $\alpha(|x'-x|,\tau)$ it is possible to represent the integral constitutive relations given by Equation (1) in an equivalent differential form as (Eringen & Edelen, 1972):

$$(1-(e_0 a)\nabla^2)\sigma_{kl} = t_{kl} \tag{4}$$

where ∇^2 is the Laplacian operator. Thus, the scale length $e_0 a$ takes into account the size effect on the response of nanostructures. For an elastic material in the one dimensional case, the nonlocal constitutive relations may be simplified as (Eringen, 1983):

$$\sigma_{xx} - \mu\frac{\partial^2\sigma_{xx}}{\partial x^2} = E\,\varepsilon_{xx} \tag{5}$$

$$\sigma_{xz} - \mu\frac{\partial^2\sigma_{xz}}{\partial x^2} = G\,\gamma_{xz} \tag{6}$$

where σ and ε are the nonlocal stress and strain respectively, $\mu = (e_0 a)^2$ is nonlocal parameter, E is the elasticity modulus, $G=E/2(1+v)$ is the shear modulus and ν is the Poisson's ratio.

2.2 Timoshenko beam theory (TBT)

Timoshenko beam theory considers the effects of shear deformation and rotational inertia of the beam. Here x-coordinate is taken along the length of the beam, z-coordinate along the thickness (the height) of the beam, and the y-coordinate is taken along the width of the beam. According to TBT, the components of displacement vector for an arbitrary point can be defined as:

$$u_1(x,z,t) = u(x,t) - z\varphi(x,t) \tag{7a}$$

$$u_2(x,z,t) = 0 \tag{7b}$$

$$u_3(x,z,t) = w(x,t) \tag{7c}$$

where u, w and φ are the axial, transverse and angular displacement along the midplane of the beam respectively. The nonzero strains of the TBT are obtained as:

$$\varepsilon_{xx} = \varepsilon_{xx}^0 + z\kappa_x, \qquad \varepsilon_{xx}^0 = \frac{\partial u(x,t)}{\partial x}, \qquad \kappa_x = \frac{\partial \varphi(x,t)}{\partial x} \tag{8a}$$

$$\gamma_{xz} = \frac{\partial w(x,t)}{\partial x} + \varphi(x,t) \tag{8b}$$

The governing equations of motion and the edge conditions based on TBT can be derived by Hamilton's principles as follows:

$$\int_0^t \delta(T - U + V)dt = 0 \tag{9}$$

where U is the strain energy, T is the kinetic energy and V is work done by external forces. The first variation of the strain energy can be calculated as:

$$\delta U = \int_v \sigma_{ij}\delta\varepsilon_{ij} dV = \int_v (\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{xz}\delta\gamma_{xz})dV \tag{10}$$

Substituting Equation (8) into Equation (10) yields:

$$\delta U = \int_0^L (N(\delta\varepsilon_{xx}^0) + M_x(\delta\kappa_x) + Q(\delta\gamma_{xz}))dx \tag{11}$$

where N, M and Q are the axial force, bending moment and shear force, respectively which are defined as

$$N = \int_A \sigma_{xx} dA, \qquad M = \int_A \sigma_{xx} z dA, \qquad Q = \int_A K_s \sigma_{xz} dA \tag{12}$$

Here K_s denotes the shear correction factor. Moreover the first variation of kinetic energy and the work done by external forces for TBT can be calculated as:

$$\delta T = \int_0^L \left[\rho A \frac{\partial u}{\partial t}\delta(\frac{\partial u}{\partial t}) + \rho I \frac{\partial \varphi}{\partial t}\delta(\frac{\partial \varphi}{\partial t}) + \rho A \frac{\partial w}{\partial t}\delta(\frac{\partial w}{\partial t}) \right] dx \tag{13}$$

$$\delta V = \int_0^L \left(f\,\delta u + q\,\delta w\right) dx \tag{14}$$

Substituting Equations (12), (13) and (14) into Equation (9) and setting the coefficients of δu, δw and $\delta\varphi$ to zero, leads to the following motion equations:

$$\frac{\partial N}{\partial x} + f = \rho A \frac{\partial^2 u}{\partial t^2} \tag{15}$$

$$\frac{\partial Q}{\partial x}+q=\rho A\,\frac{\partial^2 w}{\partial t^2} \tag{16}$$

$$\frac{\partial M}{\partial x}-Q=\rho I\,\frac{\partial^2 \varphi}{\partial t^2} \tag{17}$$

while the corresponding boundry conditions are given as:

$$N=0 \text{ or } u=0 \quad \text{at } x=0 \text{ and } x=L \tag{18}$$

$$Q=0 \text{ or } w=0 \quad \text{at } x=0 \text{ and } x=L \tag{19}$$

$$M=0 \text{ or } \varphi=0 \quad \text{at } x=0 \text{ and } x=L \tag{20}$$

Further, nonlocal axial normal force, bending moment and shear force can be obtained as:

$$N-\mu\frac{\partial^2 N}{\partial x^2}=EA\,\frac{\partial u}{\partial x} \tag{21}$$

$$M-\mu\frac{\partial^2 M}{\partial x^2}=EI\,\frac{\partial \varphi}{\partial x} \tag{22}$$

$$Q-\mu\frac{\partial^2 Q}{\partial x^2}=GAK_s\,(\frac{\partial w}{\partial x}+\varphi) \tag{23}$$

Thus the nonlocal governing equations for transverse vibrations of Timoshenko nanobeam can be derived in terms of the displacements by substituting Equations (22) and (23), into Equations (16) and (17) as follows:

$$K_sGA\,(\frac{\partial^2 w}{\partial x^2}+\frac{\partial \varphi}{\partial x})+\mu(\rho A\,\frac{\partial^4 w}{\partial t^2\partial x^2}-\frac{\partial^2 q}{\partial x^2})+q=\rho A\,\frac{\partial^2 w}{\partial t^2} \tag{24}$$

$$EI\,\frac{\partial^2 \varphi}{\partial x^2}-K_sGA\,(\frac{\partial w}{\partial x}+\varphi)+\mu\rho I\,\frac{\partial^4 \varphi}{\partial t^2\partial x^2}=\rho I\,\frac{\partial^2 \varphi}{\partial t^2} \tag{25}$$

DIFFERENTIAL TRANSFORMATION METHOD

Differential transformation method is one of the novel techniques to solve the differential equations with small calculation errors and ability to solve nonlinear equations with edge conditions value problems. Abdel-Halim Hassan (2002) applied the DTM on eigenvalues and normalized eigenfunctions. Also Wang (2013) presented the axial vibration analysis of stepped bars utilizing DTM. DTM is proved to be a good computational tool for various engineering problems. Using DTM, the ordinary and partial differential equations can be transformed into algebraic equations, from which a closed-form series solution can be obtained easily. In this method, certain transformation rules are applied to both the governing differential equations of motion and the boundary conditions of the system in order to transform them into a set of algebraic equations as presented in Tables 1 and 2. The solution of these algebraic equations gives the desired results of the problem. The basic definitions and the application procedure of this method can be introduced as follows. The transformation equation of function *f*(*x*) can be defined as (Chen and Ju, 2004):

$$F\left[k\right]=\frac{1}{k!}\left(\frac{d^k f(x)}{dx^k}\right)_{x=x_0} \tag{26}$$

where *f*(*x*) the original is function and *F*[*k*] is the transformed function. The inverse transformation is defined as

$$f\left(x\right)=\sum_{k=0}^{\infty}(x-x_0)^k F\left[k\right] \tag{27}$$

Combining Equations (26) and (27) one obtains

$$f\left(x\right)=\sum_{k=0}^{\infty}\frac{(x-x_0)^k}{k!}\left(\frac{d^k f\left(x\right)}{dx^k}\right)_{x=x_0} \tag{28}$$

In actual application, the function *f*(*x*) is expressed by a finite series and Equation (28) can be written as follows

$$f\left(x\right)=\sum_{k=0}^{N}\frac{(x-x_0)^k}{k!}\left(\frac{d^k f\left(x\right)}{dx^k}\right)_{x=x_0} \tag{29}$$

which implies that the following terms in relation (29) is negligible

$$f\left(x\right)=\sum_{k=N+1}^{\infty}\frac{(x-x_0)^k}{k!}\left(\frac{d^k f\left(x\right)}{dx^k}\right)_{x=x_0} \tag{30}$$

Table 1. Some of transformation rules for the one-dimensional DTM (Chen & Ju, 2004)

Original function	**Transformed function**
$f\left(x\right)=g\left(x\right)\pm h(x)$	$F\left(K\right)=G\left(K\right)\pm H(K)$
$f\left(x\right)=\lambda g(x)$	$F\left(K\right)=\lambda G(K)$
$f\left(x\right)=g\left(x\right)h(x)$	$F\left(K\right)=\sum_{l=0}^{K}G\left(K-l\right)H(l)$
$f\left(x\right)=\frac{d^n g(x)}{dx^n}$	$F\left(K\right)=\frac{\left(k+n\right)!}{k!}G(K+n)$
$f\left(x\right)=x^n$	$F\left(K\right)=\delta\left(K-n\right)=\begin{cases}1 & k=n\\ 0 & k\neq n\end{cases}$

Table 2. Transformed boundary conditions (B.C.) based on DTM (Chen and Ju, 2004)

X=0		**X=L**	
Original B.C.	**Transformed B.C.**	**Original B.C.**	**Transformed B.C.**
$\mathrm{f}(0)=0$	$\mathrm{F}[0]=0$	$\mathrm{f}(L)=0$	$\sum_{k=0}^{\infty}F[k]=0$

$\frac{\mathrm{df}(0)}{\mathrm{dx}}=0$	$\mathrm{F}[1]=0$	$\frac{\mathrm{df}(L)}{\mathrm{dx}}=0$	$\sum_{k=0}^{\infty} k\ F[k]=0$
$\frac{d^2 f\ (0)}{\mathrm{dx}^2}=0$	$F[2]=0$	$\frac{d^2 f\ (L)}{\mathrm{dx}^2}=0$	$\sum_{k=0}^{\infty} k\ (k-1) F[k]=0$
$\frac{d^3 f\ (0)}{\mathrm{dx}^3}=0$	$F[3]=0$	$\frac{d^3 f\ (L)}{\mathrm{dx}^3}=0$	$\sum_{k=0}^{\infty} k\ (k-1)(k-2) F[k]=0$

Using the transformation rules described in Table 1 and Equations (24) and (25) for TBT, the governing equation for nanobeam in DTM form can be expressed as:

$$(K_s GA-\mu\rho A\,\omega^2)(k+1)(k+2)\ W[k+2]+K_s GA\,(k+1)\ \phi[k+1]+\rho A\,\omega^2\ W[k]=0 \qquad (31)$$

$$(EI-\mu\rho I\,\omega^2)(k+1)(k+2)\ \phi[k+2]-K_s GA\,(k+1)\ W[k+1]+(\rho I\,\omega^2-K_s GA)\ \phi[k]=0 \qquad (32)$$

where $W[k]$, $\phi[k]$ are the transformed functions of w and φ respectively. Hence, edge conditions using the Table 2 and Equations (18) to (20) for TBT can be obtained as:

- Simply supported–Simply supported:

$W\ [0]=0\ ,\ \phi[1]=0$

$$\sum_{k=0}^{\infty} W\ [k]=0\ ,\ \sum_{k=0}^{\infty} k\ \ \phi[k]=0 \qquad (33a)$$

- Clamped–Clamped:

$\mathrm{W}[0]=0\ ,\ \phi[0]=0$

$$\sum_{k=0}^{\infty} W\ [k]=0\ ,\ \sum_{k=0}^{\infty} \phi[k]=0 \qquad (33b)$$

- Clamped–Simply supported:

$\mathrm{W}[0]=0\ ,\ \phi[0]=0$

$$\sum_{k=0}^{\infty} W\ [k]=0\ ,\ \sum_{k=0}^{\infty} k\ \ \phi[k]=0 \qquad (33c)$$

- Clamped-Free:

$\mathrm{W}[0]=0\ ,\ \phi[0]=0$

$$\sum_{k=0}^{\infty} k\ \ \phi[k]=0\ ,\ \sum_{k=0}^{\infty} (\phi\ \ [k]+k\ \ W\ [k])=0 \qquad (33d)$$

Finally, by using Equations (31) and (32) for TBT with the transformed boundary conditions one arrives at the following eigenvalue problem

$$\begin{bmatrix} A_{11}(\omega) & A_{12}(\omega) \\ A_{21}(\omega) & A_{22}(\omega) \end{bmatrix} [C]=0 \qquad \mathbf{(34)}$$

where $[C]$ corresponds to the missing edge condition at $x = 0$. For the non-trivial solutions of Equation (34), it is necessary that the determinant of the coefficient matrix is equal to zero

$$\begin{vmatrix} A_{11}(\omega) & A_{12}(\omega) \\ A_{21}(\omega) & A_{22}(\omega) \end{vmatrix} = 0 \quad \textbf{(35)}$$

Solution of Equation (35) is simply a polynomial root finding problem. Many techniques such as Newton's method, Laguerre's method, etc. can be used to find the roots of this frequency equation. Also the non-dimensional natural frequencies are obtained based on the following relation:

$$\hat{\omega} = \omega L^2 \sqrt{\rho A / EI} \quad \textbf{(36)}$$

NUMERICAL RESULTS AND DISCUSSIONS

In this section, accuracy and efficiency of the presented method and closed form solution for natural frequency are investigated through examples. For this purpose, nanobeam with the following properties are used in computing the numerical values (Thai, 2012):

$$L = 10nm, \quad \nu = 0.3, \quad \rho = 1, \quad I = bh^3/12, \quad E = 30*10^6, \quad K_s = 5/6 \quad \textbf{(36)}$$

The convergence study for natural frequency of nanobeams with various edge conditions is presnted in Table 3. It is seen that the first frequency for TBT, generally, converges at 20th iteration. Therefore, number of iteration is selected as $k = 20$ for results reported herein for the first natural frequency. Moreover, Figure 1 depicts the convergence rate for fundamental frequencies of Timoshenko nanobeam with various edge conditions.

Table 3. Convergence of fundamental frequencies for different edge conditions ($L/h = 10$, $\mu = 0$)

k	Edge Condition			
	S-S	C-C	C-S	C-F
8	9.3283	-	-	3.5708
9	9.5190	-	13.0008	3.5761
10	9.7506	-	15.2915	3.5107
11	9.7272	21.9500	14.9865	3.5210
12	9.7047	21.0527	14.9054	3.5212
13	9.7062	20.7025	14.7938	3.5211
14	9.7076	21.0254	14.8379	3.5211
15	9.7075	20.9817	14.8371	3.5211
16	9.7074	20.9730	14.8365	3.5211
17	9.7074	20.9704	14.8359	3.5211
18	9.7074	20.9725	14.8361	3.5211
19	9.7074	20.9723	14.8361	3.5211
20	9.7074	20.9723	14.8361	3.5211

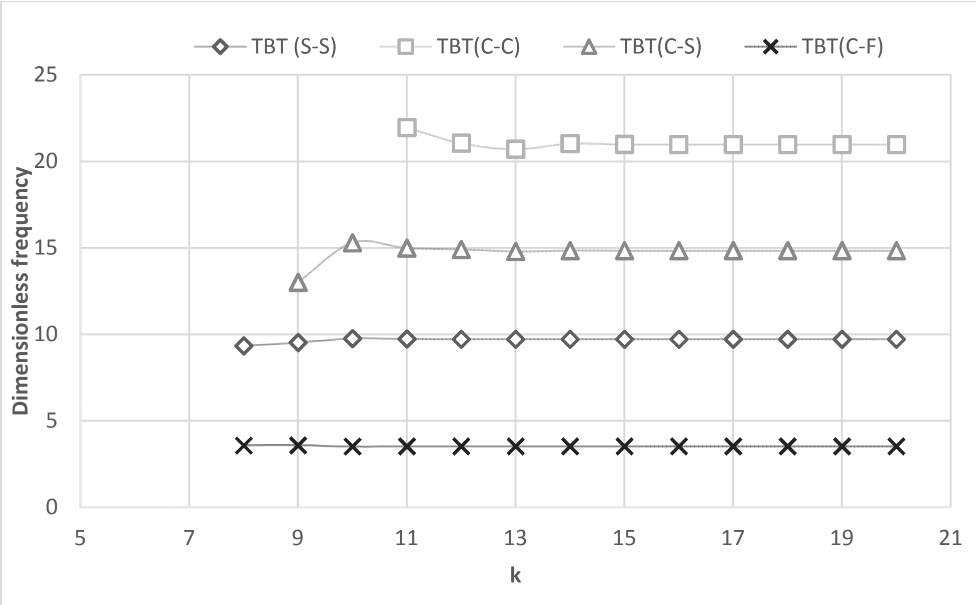

Figure 1. Convergence study of fundamental frequencies of Timoshenko nanobeam with various edge conditions (L/h = 10 , μ = 0)

In Table 4, natural frequency of nanobeam based on DTM are compared with those reported by Thai (2012). As can be seen in Table 4, the good agreement and a close correlation among the results validate the proposed method of solution.

Table 4. Comparison of non-dimensional fundamental frequencies for S-S nanobeams

L/h	$\mu=(e_0a)^2$	**Thai (2012)**	**Present (DTM)**
5	0	9.2740	9.27403971
	1	8.8477	8.84769556
	2	8.4752	8.47521568
	3	8.1461	8.14613652
	4	7.8526	7.85263561
10	0	9.7075	9.70747723
	1	9.2612	9.26120719
	2	8.8713	8.87131884
	3	8.5269	8.52685963
	4	8.2196	8.21964147
20	0	9.8281	9.82812715
	1	9.3763	9.37631061
	2	8.9816	8.98157652
	3	8.6328	8.63283617
	4	8.3218	8.32179974
100	0	9.8679	9.86793274
	1	9.4143	9.41428627
	2	9.0180	9.01795343
	3	8.6678	8.66780063
	4	8.3555	8.35550445

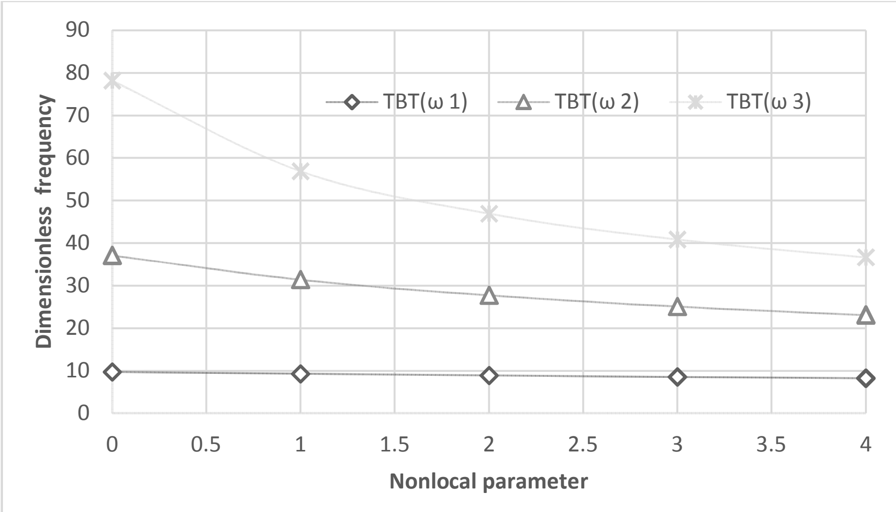

Figure 2. Non-dimensional natural frequencies of S-S nanobeams (L/h = 10)

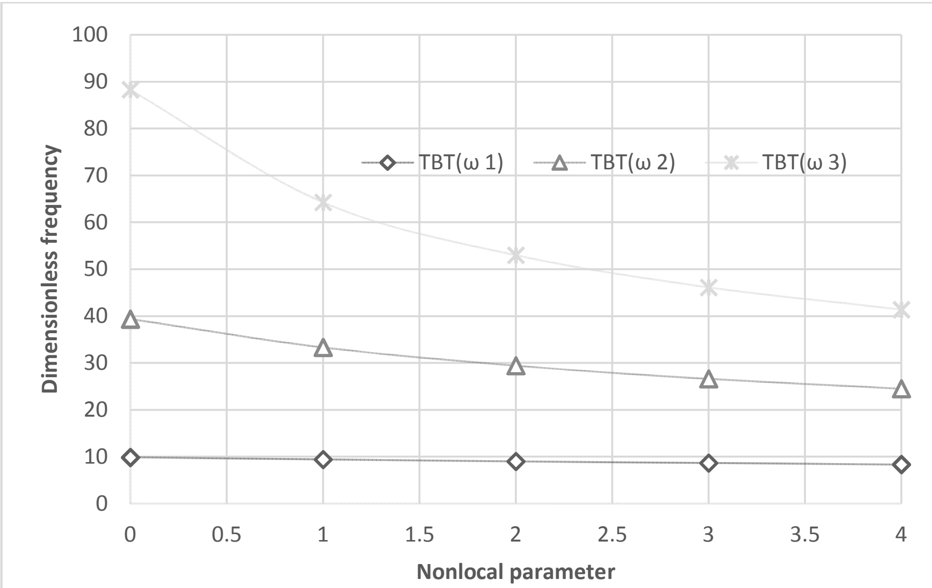

Figure 3. Non-dimensional fundamental frequencies of S-S nanobeams (L/h = 50)

The natural frequencies of a nanobeam with various edge conditions based on TBT is presented in Tables 5 and 6 for different values of aspect ratios and nonlocal parameters $\mu = (e_0a)^2$. The nonlocal parameters are taken as 0, 1, 2, 3, and 4 nm^2. It should be noted that μ = 0 corresponds to the local beam theory. It is found that the nonlocal parameter has a marked effect on the natural frequency. The effect of nonlocal parameter on non-dimensional frequencies of simply supported nano beams are shown in Figures 2 and 3 for two values of aspect ratios (L/h=10, 50). While Figures 4 and 5 depics the effect of nonlocal parameter on non-dimensional frequencies of nanobeams with various edge conditions for two values of aspect ratios (L/h=10, 50). It is seen that an increase in the nonlocal parameter leads to decrease of natural frequency. The reason is that the presence of the nonlocal effect tends to decrease the stiffness of the nanostructures and hence decreases the values of frequencies. Further, it can be concluded that with the increase the aspect ratio, the natural frequency increase.

Table 5. Non-dimensional natural frequencies of S-S nanobeams

L/h	$\mu = (e_0 a)^2$	$\hat{\omega}_1$	$\hat{\omega}_2$	$\hat{\omega}_3$
10	0	9.7074	37.0962	78.1547
	1	9.2612	31.4105	56.8753
	2	8.8713	27.7303	46.9034
	3	8.5268	25.0996	40.8254
	4	8.2196	23.0989	36.6272
20	0	9.8281	38.8299	85.6619
	1	9.3763	32.8786	62.3385
	2	8.9815	29.0263	51.4087
	3	8.6328	26.2727	44.7469
	4	8.3218	24.1785	40.1454
50	0	9.8629	39.3719	88.2907
	1	9.4095	33.3375	64.2516
	2	9.0133	29.4315	52.9864
	3	8.6634	26.6395	46.1201
	4	8.3512	24.5160	41.3774
100	0	9.8679	39.3719	88.2907
	1	9.4142	33.3375	64.2516
	2	9.0179	29.4315	52.9864
	3	8.6678	26.6395	46.1201
	4	8.3555	24.5160	41.3774

Table 6. Non-dimensional fundamental frequencies of nanobeams with different edge conditions

L/h	$\mu = (e_0 a)^2$	**Edge Condition**		
		C-C	**C-S**	**C-F**
10	0	20.9723	14.8361	3.5211
	1	19.8080	14.0555	3.5357
	2	18.8121	13.3841	3.5507
	3	17.9486	12.7989	3.5660
	4	17.1910	12.2831	3.5818
20	0	21.9953	15.2657	3.5173
	1	20.7582	14.4569	3.5324
	2	19.7037	13.7621	3.5478
	3	18.7916	13.1573	3.5637
	4	17.9931	12.6247	3.5800
50	0	22.3114	15.3935	3.5162
	1	21.0516	14.5761	3.5314
	2	19.9789	13.8744	3.5470
	3	19.0520	13.2637	3.5630
	4	18.2409	12.7262	3.5795
100	0	22.3578	15.4120	3.5160
	1	21.0946	14.5934	3.5313
	2	20.0193	13.8907	3.5469
	3	19.0901	13.2792	3.5629
	4	18.2772	12.7409	3.5794

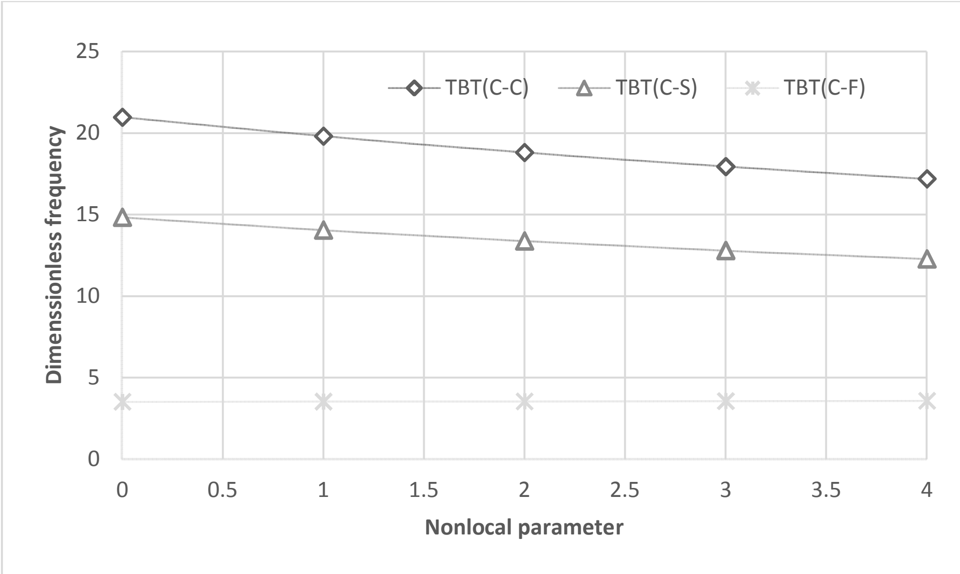

Figure 4. Non-dimensional fundamental frequencies of different edge conditions (L/h = 10)

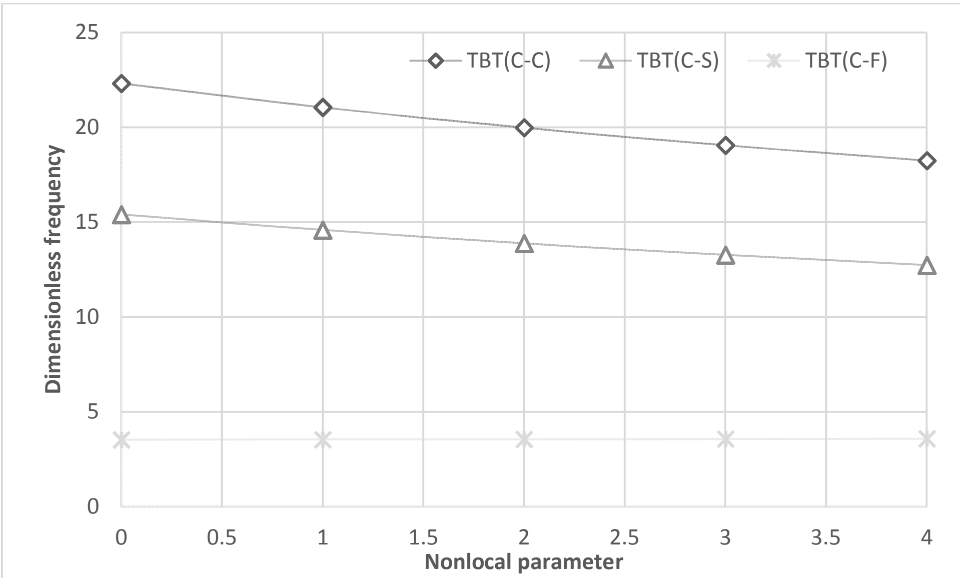

Figure 5. Non-dimensional fundamental frequencies of different edge conditions (L/h = 50)

CONCLUSIONS

The influence of nonlocal parameter effect and aspect ratio on free vibration of nanobeams with various edge condition is studied based on the Timoshenko beam theory. The equation of motion is obtained and differential transformation method is utilized in obtaining natural frequencies. It is observed that inclusion of the nonlocal effect decreases the natural frequencies of nanobeams especially at high values of nonlocal parameters while increasing the aspect ratio leads to an increase in natural frequencies. Presented numerical results can serve as benchmarks for the application and the design of nanoelectronic and nano-drive devices, nano-oscillators, and nanosensors, in which nanobeams act as basic elements.

REFERENCES

Abdel-Halim Hassan, I. H. (2002). On solving some eigenvalue problems by using a differential transformation. *Applied Mathematics and Computation*, *127*(1), 1-22.

Ansari, R. & Sahmani, S. (2012). Small scale effect on vibrational response of single-walled carbon nanotubes with different boundary conditions based on nonlocal beam models. *Communications in Nonlinear Science and Numerical Simulation*, *17*(4), 1965-1979.

Aydogdu, M. (2009). A general nonlocal beam theory: its application to nanobeam bending, buckling and vibration. *Physica E: Low-dimensional Systems and Nanostructures*, *41*(9), 1651-1655.

Chen, C. O. K. & Ju, S. P. (2004). Application of differential transformation to transient advective–dispersive transport equation. *Applied Mathematics and Computation*, *155*(1), 25-38.

Chong, A. C. M., Yang, F., Lam, D. C. C., & Tong, P. (2001). Torsion and bending of micron-scaled structures. *Journal of Materials Research*, *16*(04), 1052-1058.

Eringen, A. C. (1983). On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves. *Journal of Applied Physics*, *54*(9), 4703-4710.

Eringen, A. C. (Ed.). (2002). *Nonlocal continuum field theories*. Springer.

Eringen, A. C., & Edelen, D. G. B. (1972). On nonlocal elasticity. *International Journal of Engineering Science*, *10*(3), 233-248.

Reddy, J. N. (2007). Nonlocal theories for bending, buckling and vibration of beams. *International Journal of Engineering Science*, *45*(2), 288-307.

Thai, H. T. (2012). A nonlocal beam theory for bending, buckling, and vibration of nanobeams. *International Journal of Engineering Science*, *52*, 56-64.

Wang, Z. G. (2013). Axial vibration analysis of stepped bar by differential transformation method. *Applied Mechanics and Materials*, *419*, 273-279

Zhou, J. K. (1986). Differential transformation and its applications for electrical circuits.