OPTIMAL DESIGN OF A STEPPER-DRIVEN PLANAR LINKAGE USING ENTROPY METHODS

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ABSTRACT

Stepper motors can be used to provide open-loop motion control in mechanisms. Unlike servo controlled mechanisms, however, the rotational drive input error cannot be resolved below the step error in the motor. Therefore, there is a fixed level of rotational position error that must be accepted in stepper driven mechanisms, and this rotational position error will inevitably propagate to kinematic position error in the mechanism. In this paper, the direct linearization method will be used to derive a model for kinematic position error based on uncertainty in the rotational input angle of a mechanism. Using this model, a method of constrained optimization to design a mechanism to minimize the effect of uncertain input conditions on kinematic position will be presented. The method is based on entropy minimization techniques that have been applied in a variety of robotic system applications. The method will be demonstrated in a case study, and will be shown to optimize the positioning reliability of a mechanism under input angle errors. The method will be shown to accurately predict drive error propagation, through comparison to Monte Carlo simulation. When coupled with entropy-based system reliability optimization methods, optimal mechansims can be synthesized in response to various positioning constraints.

KEYWORDS: *Mechanisms; Optimization; Direct linearization; Entropy*

1.0 INTRODUCTION

Stepper motors offer the ability to provide precise motion control without the need for closed-loop control systems. As the motor will index a fixed rotational increment with each pulse applied to the motor, simple step-counting control methods can be used for position control. However, the positioning accuracy of a stepper motor is a physical limitation of the motor itself. Stepper motor accuracy is typically rated as a percentage of the motor step size; this accuracy is a property of

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motor construction, and cannot be increased through traditional closed-loop control methods. For example, a typical stepper motor might be rated at 1.8°/step, with an accuracy of +/- 5%. Since this motor accuracy is a function of motor design, it cannot be reduced through feedback control methods.

Since stepper motor error is inherent in the design of the motor, kinematic mechanisms driven by stepper motors are prone to positioning errors, as the actuator error propagates through the mechanism. The effect of kinematic errors on planar mechanisms has been studied by numerous researchers, including many recent efforts. Pandey and Zhang propose an entropy method for optimizing the reliability of robotic manipulator positioning (Pandey & Zhang, 2012; Zhang & Pandey, 2013). This approach uses simulation trials to evaluate the position error distribution of the manipulator under joint angle uncertainties. Approaches based on strain-energy error minimization (Aviles, et al., 2009) have been developed for optimal synthesis of planar linkages. A method based on reliability optimization using covariance matrix estimates has also been proposed (Huang & Zhang, 2010, 2012). Each of these methods can be used to provide some measure of design under uncertainty for a planar mechanism.

Methods based on direct linearization to estimate the effect of parameter variation on mechanism position provide perhaps the most promising approach to quantifying uncertainty in planar mechanisms (Wittwer et al., 2004; Leishman & Chase, 2010). In this paper, these direct linearization methods will be adapted for use in the optimal design of stepper-driven planar mechanisms. These methods will be shown to be consistent with other entropy-based approaches to kinematic optimization (Musto, 2002).

2.0 DIRECT LINEARIZATION MODEL OF A STEPPER-DRIVEN FOUR BAR LINKAGE WITH ERROR IN THE DRIVE ANGLE

Researchers have demonstrated that the direct linearization method (DLM) can be used to accurately model the effect of manufacturing tolerances on link lengths in a kinematic linkage (Wittwer et al., 2004; Leishman & Chase, 2010). In this section, the DLM will be modified to generate a model for kinematic position error based on error in the input angle. It is this input angle, which is inherent when stepper motors are used to drive mechanisms, which will be considered in the remainder of this paper.

Consider the four-bar linkage, shown in Figure 1.

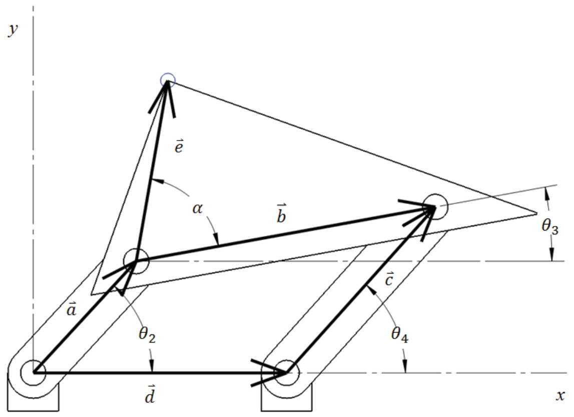

Figure 1. The four-bar linkage

The loop equation for this mechanism is:

$$\vec{a} + \vec{b} - \vec{c} - \vec{d} = 0 \tag{1}$$

This loop equation can be expanded to the following scalar equations:

$$h_1(\theta_2, \theta_3, \theta_4) = a\cos\theta_2 + \ b\cos\theta_3 - c\cos\theta_4 - d = 0 \tag{2}$$

$$h_2(\theta_2, \theta_3, \theta_4) = a\sin\theta_2 + \ b\sin\theta_3 - c\sin\theta_4 = 0 \tag{3}$$

If the coupler point is considered to be the point that represents the desired position of the mechanism, then the equations describing the position of this point can also be written in scalar form:

$$P_x(\theta_2, \theta_3, \theta_4) = a\cos\theta_2 + \ e\cos(\alpha + \theta_3) \tag{4}$$

$$P_y(\theta_2, \theta_3, \theta_4) = a\sin\theta_2 + \ e\sin(\alpha + \theta_3) \tag{5}$$

where P_x and P_y represent the x and y positions of the coupler point P.

In the positioning of the mechanism, the input angle θ_2 is uncertain; the error associated with the angle is δ_2. The error associated with θ_2 results in deviations from nominal values for θ_3 and θ_4 as well; these errors are defined as δ_3 and δ_4. Applying Equations (2) to (5) to the mechanism with uncertainty in positioning leads to the following modified equations:

$$h_1(\theta_2, \theta_3, \theta_4) = a\cos(\theta_2 + \delta_2) + b\cos(\theta_3 + \delta_3) - c\cos(\theta_4 + \delta_4) - d = 0 \tag{6}$$

$$h_2(\theta_2, \theta_3, \theta_4) = a\sin(\theta_2 + \delta_2) + b\sin(\theta_3 + \delta_3) - c\sin(\theta_4 + \delta_4) = 0 \tag{7}$$

$$P_x(\theta_2, \theta_3, \theta_4) = a\cos(\theta_2 + \delta_2) + e\cos(\alpha + \theta_3 + \delta_3) \tag{8}$$

$$P_y(\theta_2, \theta_3, \theta_4) = a\sin(\theta_2 + \delta_2) + e\sin(\alpha + \theta_3 + \delta_3) \tag{9}$$

Applying trigonometric identities, and linearizing by invoking the small angle approximation, yields:

$$h_1(\theta_2, \theta_3, \theta_4) = a\,(\cos\theta_2 - \delta_2\sin\theta_2) + b\,(\cos\theta_3 - \delta_3\sin\theta_3) - c\,(\cos\theta_4 - \delta_4\sin\theta_4) - d = 0 \tag{10}$$

$$h_2(\theta_2, \theta_3, \theta_4) = a\,(\sin\theta_2 + \delta_2\cos\theta_2) + b\,(\sin\theta_3 + \delta_3\cos\theta_3) - c\,(\sin\theta_4 + \delta_4\cos\theta_4) = 0 \tag{11}$$

$$P_x(\theta_2, \theta_3, \theta_4) = a\,(\cos\theta_2 - \delta_2\sin\theta_2) + e\cos\alpha\,(\cos\theta_3 - \delta_3\sin\theta_3) - e\sin\alpha\,(\sin\theta_3 + \delta_3\cos\theta_3) \tag{12}$$

$$P_y(\theta_2, \theta_3, \theta_4) = a\,(\sin\theta_2 + \delta_2\cos\theta_2) + e\sin\alpha\,(\cos\theta_3 - \delta_3\sin\theta_3) + e\cos\alpha\,(\sin\theta_3 + \delta_3\cos\theta_3) \tag{13}$$

Isolating the error terms only in Equations (10) and (11) yields:

$$\begin{bmatrix} -a\sin\theta_2 \\ a\cos\theta_2 \end{bmatrix}\delta_2 + \begin{bmatrix} -b\sin\theta_3 & c\sin\theta_4 \\ b\cos\theta_3 & -c\cos\theta_4 \end{bmatrix}\begin{bmatrix} \delta_3 \\ \delta_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{14}$$

which can be rewritten:

$$\boldsymbol{A}\delta_2 + \boldsymbol{B}\begin{bmatrix} \delta_3 \\ \delta_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{15}$$

or:

$$\begin{bmatrix} \delta_3 \\ \delta_4 \end{bmatrix} = \boldsymbol{B}^{-1}\boldsymbol{A}\delta_2 \tag{16}$$

This relates the values of the dependent error terms δ_3 and δ_4 in terms of the independent error δ_2, which corresponds to the position error of the driving stepper motor.

Similarly with Equations (13) and (14), the error terms can be isolated as:

$$\begin{bmatrix} \Delta P_x \\ \Delta P_y \end{bmatrix} = \begin{bmatrix} -a\sin\theta_2 \\ a\cos\theta_2 \end{bmatrix} \delta_2 + \begin{bmatrix} -e\cos\alpha\sin\theta_3 - e\sin\alpha\cos\theta_3 & 0 \\ -e\sin\alpha\sin\theta_3 + e\cos\alpha\cos\theta_3 & 0 \end{bmatrix} \begin{bmatrix} \delta_3 \\ \delta_4 \end{bmatrix} \tag{17}$$

where ΔP_x and ΔP_y are the position errors of the coupler point, or:

$$\Delta \boldsymbol{P} = \boldsymbol{C}\delta_2 + \boldsymbol{D}\begin{bmatrix} \delta_3 \\ \delta_4 \end{bmatrix} \tag{18}$$

Substituting Equation (16) into Equation (18) yields:

$$\Delta \boldsymbol{P} = (\boldsymbol{C} + \boldsymbol{D}\boldsymbol{B}^{-1}\boldsymbol{A})\ \delta_2 \tag{19}$$

which relates the position error of the coupler point to the rotational position error of the driving stepper motor. The matrix term ($\boldsymbol{C}$+$\boldsymbol{D}\boldsymbol{B}^{-1}$ **A**), which describes how the rotational error in the drive angle propagates to position error in known as the sensitivity matrix (**S**). For the typical four-bar linkage described in Figure 1, this sensitivity matrix is:

$$\boldsymbol{S} = \begin{bmatrix} -a\sin\theta_2 + \frac{ae\sin(\theta_2-\theta_4)\sin(\alpha+\theta_3)}{b\sin(\theta_3-\theta_4)} \\ a\cos\theta_2 - \frac{ae\sin(\theta_2-\theta_4)\cos(\alpha+\theta_3)}{b\sin(\theta_3-\theta_4)} \end{bmatrix} \tag{20}$$

The statistical variance of position error can be estimated using the sensitivity matrix $\boldsymbol{S}$ and the statistical properties of the rotational position error δ_2 as follows:

$$\sigma_{P_x}^2 = \left(-a\sin\theta_2 + \frac{ae\sin(\theta_2-\theta_4)\sin(\alpha+\theta_3)}{b\sin(\theta_3-\theta_4)}\right)^2 \sigma_{\delta_2}^2 \tag{21}$$

$$\sigma_{P_y}^2 = \left(a\cos\theta_2 - \frac{ae\sin(\theta_2-\theta_4)\cos(\alpha+\theta_3)}{b\sin(\theta_3-\theta_4)}\right)^2 \sigma_{\delta_2}^2 \tag{22}$$

where $\sigma_{P_x}^2$ and $\sigma_{P_y}^2$ are the variances of the coupler point position errors, and $\sigma_{\delta_2}^2$ is the variance of the rotational error in the drive angle. For stepper motors, $\sigma_{\delta_2}^2$ is readily estimated from the known properties of the motor. Stepper motors are generally rated as being accurate to a given percentage of the step size (typically 3% to 5% in a common stepper motor), yielding bidirectional tolerance bounds on stepper motor position. Conservatively, the rotational position error can be considered to be uniformly distributed over the error bounds, and the variance calculated accordingly.

In order to validate the DLM model for error propagation, a case study was used. Using the geometry as defined in Figure 1, the following dimensional values were used: a=40 mm, b=120 mm, c=80 mm, d=100 mm, e=50 mm,and α=30°. The driving stepper motor was assumed to have a step size of 1.8°/step, and an accuracy of ±5%. Under the conservative assumption of a uniformly distributed stepper motor error, a variance of $\sigma_{\delta_2}^2 = 8.127 \times 10^{-7} \text{rad}^2$ was used.

Equations (21) and (22) were used to predict position error variance as a function of the input angle θ_2 over a full revolution of the input link (in 1.8° increments). In addition, Monte Carlo simulations using 10,000 trials at each value of θ_2, and the position error variance was computed from these simulated values. Figure 2 shows the path of the coupler for one full revolution of the crank link in both the "open" and "crossed" configurations.

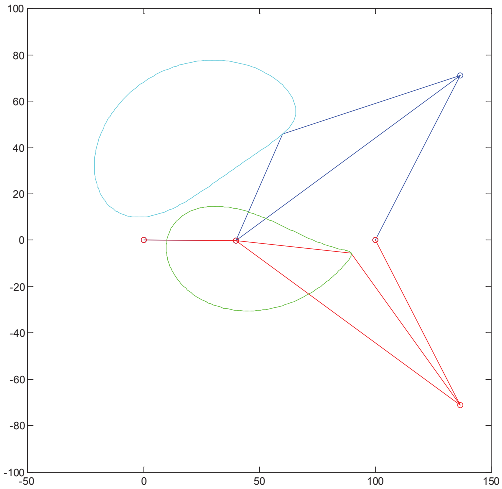

Figure 2. Coupler curves for open and crossed configurations (units of mm)

Figures 3 and 4 show the simulated vs. predicted values of $\sigma_{P_x}^2$ and $\sigma_{P_y}^2$ using the "open" configuration of the four-bar mechanism, and Figures 5 and 6 show the simulated vs. predicted values of $\sigma_{P_x}^2$ and $\sigma_{P_y}^2$ using the "crossed" configuration of the four-bar mechanism, using the known kinematic solutions (Norton, 1999). In each case, the DLM model accurately predicts the position error variance.

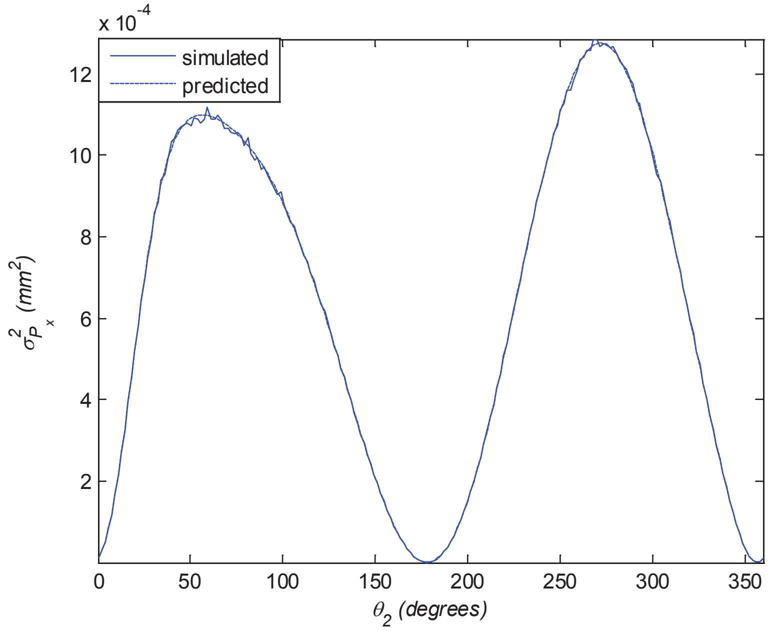

Figure 3. Variance of coupler x position (open configuration)

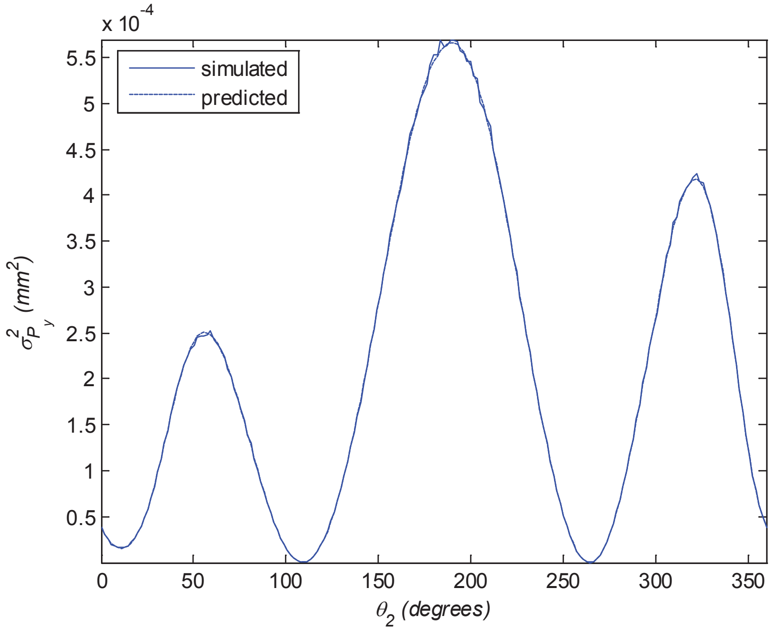

Figure 4. Variance of coupler y position (open configuration)

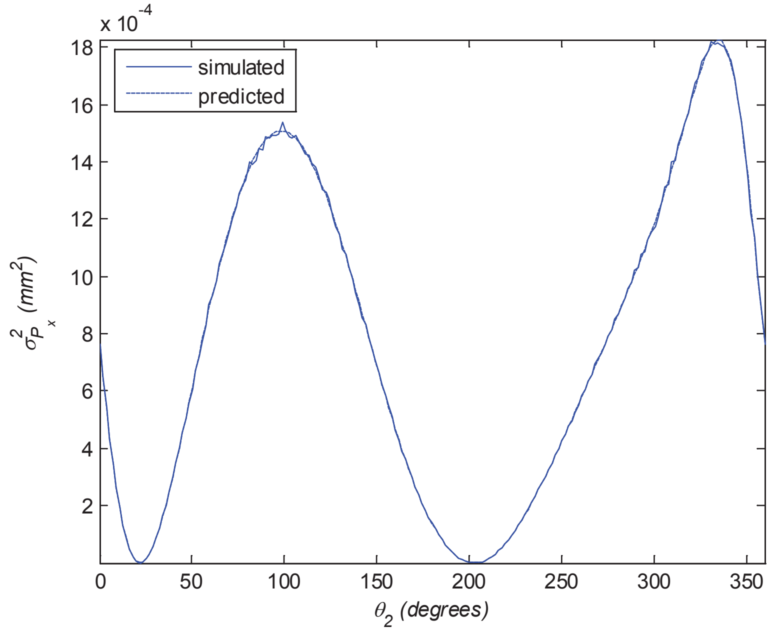

Figure 5. Variance of coupler x position (crossed configuration)

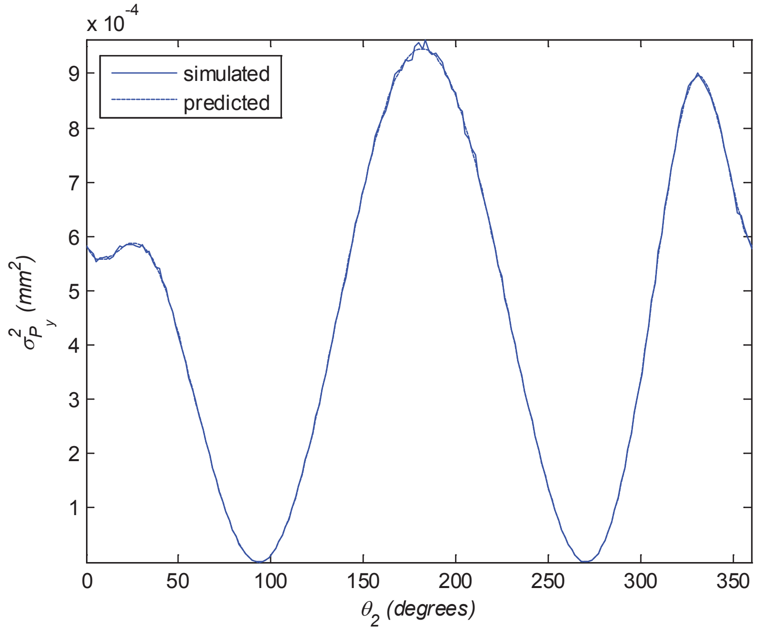

Figure 6. Variance of coupler y position (crossed configuration)

3.0 AN ENTROPY BASED APPROACH TO RELIABILITY OPTIMIZATION

In the previous section, it was demonstrated that the DLM method can be used to establish an analytic expression for position error variance in a stepper-driven four-bar mechanism. Consider the case where the four-bar mechanism is being used to precisely place the coupler point at a desired x-y location, with allowable position error described by bidirectional tolerances on the x and y components of position:

$$P_{x,allow} = P_{x,des} \pm x_{tol} \tag{23}$$

$$P_{y,allow} = P_{y,des} \pm y_{tol} \tag{24}$$

where $P_{x'allow}$ and $P_{y'allow}$ indicate the allowable values of position, $P_{x'des}$ and $P_{y'des}$ indicate the nominally desired position for the coupler point, and x_{tol} and y_{tol} indicate bidirectional tolerance constraints on the nominal desired position.

Since the position of the mechanism has been modeled probabilistically, the positioning problem can be interpreted from a reliability perspective; the optimal four-bar mechanism will maximize the probability that the coupler point falls within the tolerance bound place on position, thereby maximizing positioning reliability. This problem has been addressed for open-loop kinematic chains and robotic manipulators by employing an entropy formulation (Musto, 2002); under the assumption that the system is designed to nominally hit the desired x-y position, then the reliability of the positioning system can be expressed as:

$$R = Prob\{\Delta P_x{}^2 \leq x_{tol}^2 \cap \Delta P_y{}^2 \leq y_{tol}^2\} \tag{25}$$

where R is the reliability of the positioning system. In systems where the position error variance is known, it has been shown that minimizing an entropy measure associated with the position error is equivalent to optimizing a bound on system reliability; a lower-bound on position reliability is optimized when the following entropy-based objective function is minimized:

$$V = \frac{\sigma_{P_x}^2}{x_{tol}^2} + \frac{\sigma_{P_y}^2}{y_{tol}^2} \tag{26}$$

This approach has been demonstrated for high degree-of-freedom systems, where the error variance must be determined through Monte

Carlo simulation (Musto, 2002). However, when coupled with the DLM method for stepper-driven four-bar linkages, this objective function can be evaluated analytically, and used as an objective function for design optimization.

Substituting the expressions from Equations (21) and (22), and eliminating the constant term $\sigma_{\delta_2}^2$,yields the following cost function for minimization:

$$V = \frac{\left(-a\sin\theta_2 + \frac{ae\sin(\theta_2-\theta_4)\sin(\alpha+\theta_3)}{b\sin(\theta_3-\theta_4)}\right)^2}{x_{tol}^2} + \frac{\left(a\cos\theta_2 - \frac{ae\sin(\theta_2-\theta_4)\cos(\alpha+\theta_3)}{b\sin(\theta_3-\theta_4)}\right)^2}{y_{tol}^2} \quad (27)$$

This will be used in the formulation of the mechanism design problem as a multivariable constrained optimization problem.

4.0 AN OPTIMAL DESIGN FORMULATION

In the previous sections, an analytic expression for the position variance of a planar four-bar mechanism was determined, and a cost function analogous to system reliability was introduced. In this section, the design of step-driven four-bar mechanisms will be formulated as a constrained optimization problem; the goal will be to design the four-bar mechanism to optimize the reliability of the positioning system in the presence of uncertainty in the actuator (θ_2); in essence, the goal of the design will be to minimize the propagation of actuator error to coupler point positioning error.

In this formulation, it is assumed that all of the system uncertainty is due to the actuator error δ_2, and that the stepper resolution of the actuator is known. Geometric errors in the link length, pivot point placement, etc. are not considered; this is due to the fact that for a given mechanism, these errors are fixed, and the effect of these errors can be removed by calibration; only the actuator angle remains as a random parameter once the system is built and calibrated. Design variables include the length of the four links (*a, b, c, d*), the coupler length (*e*), the actuator angle (θ_2) and the coupler angle (α). The angle of the ground link *d* with respect to the x-axis could also be included as a design variable; this would in essence have the effect of modifying the relative values of x_{tol} and y_{tol}. In this development, the ground link will be assumed to be coincident with the *x*-axis.

With these assumptions, the multivariable constrained optimization problem can be formulated as:

Find: $a,b,c,d,e,\theta_2,\alpha$

To minimize:

$$f(a,b,c,d,e,\theta_2,\alpha) = \frac{\left(-a\sin\theta_2+\frac{ae\sin(\theta_2-\theta_4)\sin(\alpha+\theta_3)}{b\sin(\theta_3-\theta_4)}\right)^2}{x_{tol}^2} + \frac{\left(a\cos\theta_2-\frac{ae\sin(\theta_2-\theta_4)\cos(\alpha+\theta_3)}{b\sin(\theta_3-\theta_4)}\right)^2}{y_{tol}^2} \quad (28)$$

Subject to:

$$P_{x,des} - a\cos\theta_2 + e\cos(\alpha+\theta_3) = 0 \quad (29)$$

$$P_{y,des} - a\sin\theta_2 + e\sin(\alpha+\theta_3) = 0 \quad (30)$$

$$a\cos\theta_2 + b\cos\theta_3 - c\cos\theta_4 - d = 0 \quad (31)$$

$$a\sin\theta_2 + b\sin\theta_3 - c\sin\theta_4 = 0 \quad (32)$$

Equations (29) and (30) ensure that the mechanism will reach the nominal desired position; Equations (31) and (32) enforce the kinematic loop equations. It should be noted that the link lengths *a, b, c*, and *d* must be nonnegative values, which introduces additional constraints.

In the practical implementation of this method, additional constraints are likely. For example:

- Nonzero lower bounds may be placed on link lengths, to allow for practical geometric constraints such as bearing/pin sizes, etc.
- Upper bounds may be placed on link lengths, limiting the overall size envelope for the mechanism
- Constraints may be placed on the relative lengths of the links in the mechanism, allowing for practical construction and scale of the mechanism
- A constraint may be added on the orientation a specific link, if orientation is important to the positioning task (e.g. the angle of the coupler link may be specified)
- If desired, a constraint to enforce the Grashof condition and allow for full rotation of the drive link can be formulated (shortest link + longest link ≤ sum of remaining links)

Once formulated, a suitable computational algorithm can be used to solve the multivariable constrained optimization problem, and yield a mechanism design that is optimal from a reliability standpoint. The addition of practical constraints will vary according to the usage of the linkage, but should be readily incorporated with an appropriate computational algorithm. This will be demonstrated in the next section in a mechanism design case study.

5.0 CASE STUDY

The DLM model and entropy-based optimization algorithm will be demonstrated in a case study in this section. A four-bar linkage (as shown in Figure 1), with a pivot point placed at the origin, will be the assumed geometry for the case study. The mechanism will be designed to achieve the following positioning task:

- $P_{x'\,des}$=120 mm, with an allowable tolerance of ± 1 mm
- $P_{y'\,des}$=180 mm, with an allowable tolerance of ± .1 mm

In addition to the constraints inherent to the optimization problem described in the previous section, the following additional practical constraints have been placed on the mechanism design:

- The crank link length a will be at least 60% of the length of the other links in the kinematic chain
- The coupler link length e will be no longer than any other link in the kinematic chain
- The orientation of the coupler link (the quantity $\alpha+\theta_3$) will be specified to be between 70° and 80°
- The relative lengths of the kinematic links *a, b, c*, and *d* will be such that the mechanism meets Grashof's condition, with *a* being the shortest link in the kinematic chain

With these constraints in place, a graphical solution which achieved the nominal desired position within the kinematic constraints was formulated. The graphical solution, which was used as an initial condition for optimization, is summarized in Table 1.

Table 1. Initial condition (graphical solution)

Parameter	Initial condition
a	91.0 mm
b	150.0 mm
c	147.7 mm
d	150 mm
e	150 mm
α	30°
θ_2	22°
θ_3	46°
θ_4	75°

This graphical solution, which was obtained by traditional trial-and-error graphical methods, was evaluated using the objective function in Equation (28); the value of the objective function for this initial design was found to be V=2.34x10^5; additionally, use of Equations (21) and (22) yielded standard deviations on positioning error to be σP_x =.102 mm and σP_y =.043 mm.

With this as an initial solution, the optimization formulation offered in the previous section, in addition to the additional practical constraints, was used to improve the design. For the computational algorithm, the Solver tool in Microsoft Excel 2010 was used. Specifically, the GRG Nonlinear optimization algorithm was utilized. After performing the optimization, the mechanism design detailed in Table 2 resulted. For comparison, the initial conditions and values of objective function and positioning error estimates are shown as well.

Table 2. Mechanism design

Parameter	Initial condition	Final design
a	91.0 mm	69.0 mm
b	150.0 mm	115.0 mm
c	147.7 mm	163.9 mm
d	150 mm	163.9 mm
e	150 mm	163.9 mm
α	30°	-13°
θ_2	22°	22°
θ_3	46°	83°
θ_4	75°	121°
V	2.34x10^5	2.59x10^4
σ_{P_x}	.102 mm	.109 mm
σ_{P_y}	.043 mm	.010 mm

In order to validate the method, the design problem was reposed, with a change in the tolerance constraint on $P_{x'_{des}}$ for 1 mm in the previous case study to 0.01 mm in the revised case study. Starting from the same initial conditions, the mechanism detailed in Table 3 resulted.

Table 3. Mechanism design with $P_{x'des}$=1 mm, $P_{y'des}$=0.01 mm

Parameter	Initial condition	Final design
a	91.0 mm	72.2 mm
b	150.0 mm	159.1 mm
c	147.7 mm	120.5 mm
d	150 mm	120.6 mm
e	150 mm	159.1 mm
α	30°	57°
θ_2	22°	25°
θ_3	46°	13°
θ_4	75°	34°
V	2.21x10^7	2.96x10^5
σ_{P_x}	.102 mm	0.0002 mm
σ_{P_y}	.043 mm	.049 mm

In each case, the optimization algorithm achieved a reduction in the objective function; in each case, the mechanism configuration achieved objective function minimization by reducing error in the direction that was held to the tighter tolerance constraint.

A third case study was performed, with the tolerance set to 0.1 mm for both $P_{x'des}$ and $P_{y'des}$. In this case, with the error in x and y directions weighted equally in the objective function, the results shown in Table 3 were obtained.

Table 4. Mechanism Design with $P_{x'des}$=$P_{y'des}$=0.1 mm

Parameter	Initial condition	Final design
A	91.0 mm	84.6 mm
B	150.0 mm	141.0mm
C	147.7 mm	143.9 mm
D	150 mm	144.0 mm
E	150 mm	144.0 mm
α	30°	27°
θ_2	22°	31°
θ_3	46°	43°
θ_4	75°	78°
V	1.49x10^6	4.86 x10^5
σ_{P_x}	.102 mm	.054 mm
σ_{P_y}	.043 mm	.032 mm

In each of the three case studies, the following can be shown:

- The optimized results, when substituted into Equations (4) and (5), yield the nominal desired position of $P_{x'des}$=120 mm, $P_{y'des}$=180 mm. This is the consequence of the kinematic constraints in the optimization algorithm.
- In each case, the optimized mechanism results in an improved design (as indicated by the improvement in the

objective function) over the nominal graphical solution. This indicates more accurate positioning of the mechanism in the presence of drive angle error.

These case studies indicate that more the DLM method can be used to optimize the positioning reliability of stepper-driven mechanisms.

6.0 CONCLUSIONS

In this paper, the DLM method has been extended to use in the optimization of stepper-driven mechanisms. The method was shown to accurately predict drive error propagation, through comparison to Monte Carlo simulation. When coupled with entropy-based system reliability optimization methods, it was shown that optimal mechansims could be synthesized in response to various positioning constraints.

In this new method, only drive angle error was considered. This method could be augmented to include additional uncertainties in design, such as tolerances on link lengths, clearance in bearings, and planar misalignments. Work is underway in this area.

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